X3.17a **2050**

PHYS-205(17) (Kaldon-20619) Name S O L U T I O N

WMU-Summer I 2006

Exam 3A - 100,000 points + 20,000 ☆ points *Don't Forget Your Paper!*

06/15/2006•Rev.5

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions ☐☆2a ☐☆2b ☐☆2c ☐☆2e

EXAM 3 [FORM - A]

PHYS-2050 (KALDON-17)

SUMMER I 2006

WMU

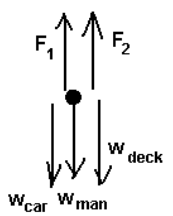
Shouldn't the game Twister be called "Torquer"?

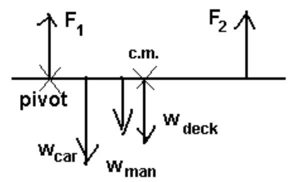
Roving the Red Planet (35,000 points)

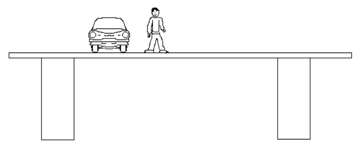
1.) (a) Three summers ago NASA sent the first of two rover probes to Mars – *Spirit* and *Opportunity* are still at work! To get to Mars, a 400. kg probe first has to leave the Earth. On Earth the probe weighs 3920 N. What would this 400. kg probe weigh on Mars (mass = 6.42 × 10^{23} kg ; radius = 3.37 × 10^{6} m)? *G = 6.67 × 10^{-11} N·m²/kg²*

$$F_G = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11}\ {}^{N\cdot m^2}/_{kg^2}\right)\left(6.42\times10^{23}\,kg\right)\left(400.\,kg\right)}{\left(3.37\times10^{6}\,m\right)^2} = 1508\,N$$

To launch a rocket to Mars, an intrepid NASA scientist/engineer first has to get to work in his car. The parking deck has a mass of 11,400 kg and is 20.0 m wide. Support Piers 1 and 2 are located 3.00 m from the ends. The 1950 kg car is 6.00 m from the left and our 1320 N scientist/engineer is 3.00 meters from the center-of-mass of the car as shown. Find the support forces (b) F_1 and (c) F_2. *To get full credit, you must include the F.B.D. and the F.R.D.*







F_1 is 0 meters from pivot.
Car is 3.00 meters from pivot.
Man is 6.00 meters from pivot.
Deck c.m. is 7.00 meters from pivot.
F_2 is 14.0 meters from pivot.

$$w_{car} = m_{car} g = (1950 kg)(9.81 m/s^2) = 19{,}130 N$$
$$w_{man} = 1320 N$$
$$w_{deck} = m_{deck} g = (11{,}400 kg)(9.81 m/s^2) = 11{,}800 N$$
$$\sum F_y = F_1 + F_2 - w_{car} - w_{man} - w_{deck} = 0$$
$$\sum \tau = F_2(14.0m) - w_{car}(3.00m) - w_{man}(6.00m) - w_{deck}(7.00m) = 0$$
$$F_2(14.0m) = w_{car}(3.00m) + w_{man}(6.00m) + w_{deck}(7.00m)$$
$$F_2 = \frac{(19{,}130N)(3.00m) + (1320N)(6.00m) + (11{,}800N)(7.00m)}{14.0m}$$
$$= 10{,}570 N$$
$$F_1 = w_{car} + w_{man} + w_{deck} - F_2$$
$$= 19{,}130N + 1320N + 11{,}800N - 10{,}570N$$
$$= 21{,}680 N$$

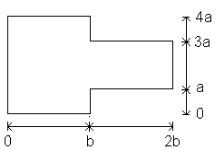
(d) Our scientist/engineer thinks he's got a powerful car – it's got 245 hp (182,800 W). If he wants to get his car from rest up to 60.0 mph (26.8 m/s), how much time would this take, assuming he can use all that power?

$$W = \Delta K = K_f - K_i = \tfrac{1}{2} m v^2$$
$$= \tfrac{1}{2}(1950 kg)(26.8 m/s)^2 = 700{,}300 J$$
$$P = \frac{W}{t}$$
$$t = \frac{W}{P} = \frac{700{,}300 J}{182{,}800 W} = 3.831 \text{ sec}$$

(e) When our scientist/engineer gets to his destination, he pulls on the parking brake with a force of 127 N. The parking brake has a 1.00 m length of cable (Y = 10.0×10^{10} N/m² , cross-section A = 9.00×10^{-6} m²). How much does the cable stretch?

$$Y = \frac{F/A}{\Delta L / L_0} = \frac{F L_0}{A \Delta L}$$
$$\Delta L = \frac{F L_0}{A Y} = \frac{(127 N)(1.00 m)}{(9.00\times10^{-6} m^2)(10.0\times10^{10} N/m^2)}$$
$$= 0.0001411 m = 1.411\times10^{-4} m$$

A Notch Above the Usual (30,000 points)



2.) ☆(a) A plate of mass m has sides of $4a$ and $2b$. Find the center of mass coordinate x_{cm} by integrating $x_{cm} = \frac{1}{M}\int x\,dm$, using the x- and y-axes as shown.

Mass on the left side should be twice the mass of the right side, since it has twice the area.

$$left:\ \lambda_L = M_L/b = \left(\tfrac{2}{3}M/b\right)\ ;\ dm = \lambda_L dx$$

$$right:\ \lambda_R = M_R/b = \left(\tfrac{1}{3}M/b\right)\ ;\ dm = \lambda_R dx$$

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_{left} x\,dm + \frac{1}{M}\int_{right} x\,dm$$

$$= \frac{1}{M}\int_0^b x\lambda_L dx + \frac{1}{M}\int_b^{2b} x\lambda_R dx$$

$$= \frac{\lambda_L}{M}\int_0^b x\,dx + \frac{\lambda_R}{M}\int_b^{2b} x\,dx$$

$$= \frac{\lambda_L}{M}\left.\frac{x^2}{2}\right|_0^b + \frac{\lambda_R}{M}\left.\frac{x^2}{2}\right|_b^{2b}$$

$$= \frac{\lambda_L}{M}\frac{b^2}{2} + \frac{\lambda_R}{M}\frac{\left(4b^2 - b^2\right)}{2}$$

$$= \frac{\left(\frac{2}{3}M/b\right)}{M}\frac{b^2}{2} + \frac{\left(\frac{1}{3}M/b\right)}{M}\frac{\left(4b^2 - b^2\right)}{2}$$

$$= \frac{b}{3} + \frac{b}{2} = \frac{2b}{6} + \frac{3b}{6} = \frac{5b}{6} = 0.8333b$$

OR

$$A = (4a)(b) + (2a)(b) = 6ab$$

$$\sigma = \frac{M}{A} = \frac{M}{6ab}$$

$$dm = \sigma\,dA = \sigma\,dxdy$$

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int x\,\sigma\,dxdy$$

$$= \frac{\sigma}{M}\left[\int_0^b\int_0^{4a} x\,dxdy + \int_b^{2b}\int_a^{3a} x\,dxdy\right]$$

$$= \frac{\sigma}{M}\left[\int_0^b x\,dx\int_0^{4a} dy + \int_b^{2b} x\,dx\int_a^{3a} dy\right]$$

$$= \frac{\sigma}{M}\left[\left.\frac{x^2}{2}\right|_0^b (4a) + \left.\frac{x^2}{2}\right|_b^{2b} (2a)\right]$$

$$= \frac{\sigma}{M}\left[\frac{b^2}{2}(4a) + \frac{4b^2 - b^2}{2}(2a)\right]$$

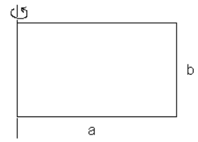
$$= \frac{\sigma}{M}\left[\frac{b^2}{2}(4a) + \frac{3b^2}{2}(2a)\right]$$

$$= \frac{\sigma}{M}\left[2ab^2 + 3ab^2\right]$$

$$= \frac{\sigma}{M}\left[5ab^2\right]$$

$$= \frac{1}{M}\left(\frac{M}{6ab}\right)\left[5ab^2\right]$$

$$= \frac{5b}{6} = 0.8333b$$



☆(b) A plate of mass m has sides of a and b. Find the moment of inertia I of the plate about the y-axis as shown, by integrating $I = \int r^2 dm$.

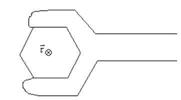
$$\lambda = \frac{M}{a}\ ;\ dm = \lambda dx\ ;\ r \to x$$

$$I = \int r^2 dm = \int_0^a x^2\,\lambda dx = \lambda\int_0^a x^2 dx$$

$$= \lambda\left[\frac{x^3}{3}\right]_0^a = \lambda\left(\frac{a^3}{3}\right)$$

$$= \left(\frac{M/a}{3}\right)\left(a^3\right) = \tfrac{1}{3}Ma^2$$

☆(c) A torque $\vec{\tau}$ to tighten a bolt consists of a force being applied at a distance from the axis of rotation. As the bolt gets tighter, it gets harder and harder to turn the bolt, so the torque as a function of angle is given by $\tau = C\,\theta^2$, where C is some constant with appropriate units. If the total work done by applying this torque through two complete revolutions is 1500. J, then find C.



$$W = \int \tau\,d\theta = \int_0^{4\pi} C\theta^2 d\theta$$

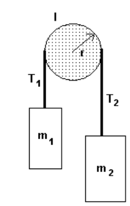
$$= C\left.\frac{\theta^3}{3}\right|_0^{4\pi} = \frac{C}{3}\left(\left(4\pi\,rad\right)^3 - 0\right)$$

$$= C\left(661.5 rad^3\right) = 1500.J$$

$$C = \frac{1500.J}{\left(661.5 rad^3\right)} = 2.267\ J/rad^3\ \ OR\ \ 2.267\ N\cdot m/rad^2$$

Those quasi-units are slippery!

(d) In the diagram at right, the mass m_1 is moving *down*. Does $\vec{\tau}$ on the pulley point *up, down, left, right, in, out*?



Rotation is counter-clockwise, so using RHR, the thumb points OUT.

The Torque vector $\vec{\tau}$ points OUT of the page.

☆(e) An solid disk of mass 71.3 kg and radius 1.21 m has a motion that follows the following equation. Find the angular acceleration, α, at time t = 1.00 sec.

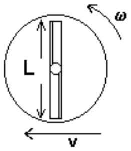
$$\omega(t) = (2.00\ rad/s) + (2.00\ rad/s^2)\,t - (2.00\ rad/s^3)\,t^2$$

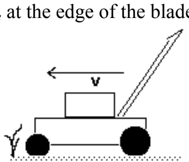
$$\omega(t) = (2.00\ rad/s) + (2.00\ rad/s^2)\,t - (2.00\ rad/s^3)\,t^2$$
$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$
$$\alpha = \frac{d}{dt}\left[(2.00\ rad/s) + (2.00\ rad/s^2)\,t - (2.00\ rad/s^3)\,t^2\right]$$
$$= 0 + (2.00 rad/s^2) - 2(2.00 rad/s^3)\,t$$
$$= (2.00 rad/s^2) - (4.00 rad/s^3)\,t$$
$$\alpha\big|_{t=1.00\sec} = (2.00 rad/s^2) - (4.00 rad/s^3)(1.00\sec)$$
$$= 2.00 rad/s^2 - 4.000 rad/s^2$$
$$= -2.000\,rad/s^2$$

If It's Green and Can Be Mowed, It's a Lawn (35,000 points)

3.) Dr. Phil's 1994 Sears self-propelled gas lawnmower has a 5.55 hp (4140 W)*Eager One* engine. The claim in an ad is that "the cutting bar strikes each blade of grass five times as the mower moves forward at normal walking speed (1.00 m/s)". This works out to the blade spinning 2.00 revolutions in 0.700 sec.

(a) What is the angular velocity ω of the blade? Given that the blade has a length L = 0.700 m, what is the linear speed v_{blade} at the edge of the blade? *Note this is not the same speed v of the mower going forward.*





$$\omega = \frac{2.00 revs}{1.00\sec} = 4\pi\, rad/\sec$$
$$D = 0.700m\ ;\ r = D/2 = 0.350m$$
$$v = r\omega = (0.350m)(4\pi\, rad/\sec)$$
$$= 1.400\pi\ m\cdot rad/\sec$$
$$= 4.398 m/s$$

(b) The ad continues, "to meet Federal safety standards, the blade brake will stop the blade in one revolution". Find the angular acceleration α needed to stop the blade. *If you didn't get an answer to (a) use ω_0 = 5.00 rad/sec.*

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$
$$0 = \omega_0^2 + 2\alpha(2\pi\, rad)$$
$$2\alpha(2\pi\, rad) = -\omega_0^2$$
$$\alpha = \frac{-\omega_0^2}{4\pi\, rad} = \frac{-(4\pi\, rad/\sec)^2}{4\pi\, rad}$$
$$= -4\pi\, rad/\sec^2 = -12.57 rad/\sec^2$$

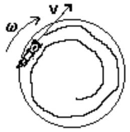
(c) The 2.00 kg blade can be modeled as a solid rod or bar. Find the torque needed to stop the blade.

$$I = \tfrac{1}{12} ML^2 = \tfrac{1}{12}(2.00kg)(0.700m)^2$$
$$= 0.08167 kg\cdot m^2$$
$$\tau = I\alpha = (0.08167 kg\cdot m^2)(-12.57\, rad/\sec^2)$$
$$= -1.027\, N\cdot m$$

(d) What is the total kinetic energy, angular and linear, of the 35.0 kg mower rolling forward at 1.00 m/s, all four wheels turning (each solid wheel is 0.500 kg and 12.0 cm (0.120 m) in diameter), blade whirring?

$$I_{wheels} = \tfrac{1}{2} MR^2 = \tfrac{1}{2}(0.500kg)(0.0600m)^2$$
$$= 0.0009000 kg\cdot m^2$$
$$K = K_{forward} + 4K_{wheels} + K_{blade}$$
$$= \tfrac{1}{2} mv^2 + 4\left(\tfrac{1}{2} I_{wheels}\omega_{wheels}^2\right) + \tfrac{1}{2} I_{blade}\omega_{blade}^2$$
$$= \tfrac{1}{2}(35.0kg)(1.00m/s)^2 + 4\left(\tfrac{1}{2}(0.0009000 kg\cdot m^2)\left(\frac{1.00m/s}{0.0600m}\right)^2\right)$$
$$+ \tfrac{1}{2}(0.08167 kg\cdot m^2)(4\pi\, rad/\sec)^2$$
$$= 17.50J + 0.5000J + 6.448J$$
$$= 24.45J$$

(e) As you mow around this circular yard, slowly spiraling towards the center, what happens to "v" and "ω" of the mower? Why? *If you aren't sure what this question is about – make sure you ask Dr. Phil!*



The linear speed of the mower "v" remains constant.

The angular velocity of the mower "ω" is going to get larger, because $\omega = v / r$, and r is getting smaller.