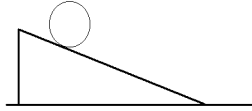


**"It's 12:34 am – do you know where your next test question is?" (50,000 points)**

1.) A solid ball of mass 0.838 kg and radius 0.135 m starts out at rest on an inclined plane 0.400 m above the ground. (a) Find the speed of the ball at the bottom of the ramp if there is no friction and the ball slides without rotating.



Easiest method is conservation of energy...

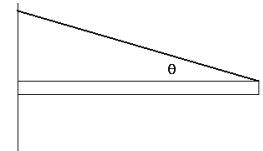
$$\begin{aligned} mgh_1 + \frac{1}{2}mv_1^2 &= mgh_2 + \frac{1}{2}mv_2^2 \\ mgh_1 &= \frac{1}{2}mv_2^2 \quad ; \quad gh_1 = \frac{1}{2}v_2^2 \\ v_2 &= \sqrt{2gh_1} = \sqrt{2(9.81\text{m/s}^2)(0.400\text{m})} \\ &= 2.801\text{m/s} \end{aligned}$$

(b) Find the speed of the ball at the bottom of the ramp if there is friction and the ball rotates without sliding.

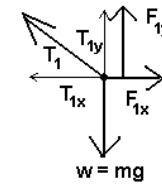
$$\begin{aligned} I_{\text{solid ball}} &= \frac{2}{5}MR^2 \quad ; \quad \omega = \frac{v}{r} \\ mgh_1 + \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 &= mgh_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 \\ mgh_1 &= \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{5}\right)(mr^2)\left(\frac{v}{r}\right)^2 \\ mgh_1 &= \frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{mr^2v^2}{r^2}\right) \\ gh_1 &= \frac{1}{2}v_2^2 + \frac{1}{5}v_2^2 = \frac{5}{10}v_2^2 + \frac{2}{10}v_2^2 = \frac{7}{10}v_2^2 \\ v_2 &= \sqrt{\frac{10}{7}gh_1} = \sqrt{\frac{10}{7}(9.81\text{m/s}^2)(0.400\text{m})} \\ &= 2.368\text{m/s} \end{aligned}$$

NOTE: 2.368 m/s < 2.801 m/s as expected.

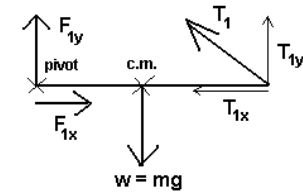
(c) A metal pole 4.00 meters long and a weight 98.1 N is kept from falling by a taut steel cable at an angle  $\theta = 24^\circ$ . Draw the Free Body Diagram and the Free Rotation Diagram of the metal pole. There is an unknown force  $F_f$  from the wall on the base of the metal pole at left – set your axis of rotation there.



F . B . D .



F . R . D .



(d) Find the tension  $T_1$  in the steel cable.

$$\begin{aligned} \sum \tau &= T_{1y}L - w\frac{L}{2} = 0 \\ T_{1y} &= \frac{w}{2} = T_1 \sin \theta \\ T_1 &= \frac{w}{2 \sin \theta} = \frac{(98.1\text{N})}{2(\sin 24^\circ)} = 120.6\text{N} \end{aligned}$$

(e) The steel cable accidentally snaps. Find the initial angular acceleration  $\alpha$  of the metal pole as it begins to freely rotate about the left end of the pole. Again, the unknown force  $F_1$  from the wall is at the axis of rotation, so it doesn't enter into this calculation.

$$w = mg ; m = \frac{w}{g} = \frac{(98.1N)}{(9.81m/s^2)} = 10.00kg$$

$$I_{rod, end} = \frac{1}{3}mL^2 = \frac{1}{3}(10.00kg)(4.00m)^2 = 53.33kg \cdot m^2$$

$$\sum \tau = w \frac{L}{2} = I\alpha$$

$$\alpha = \frac{wL}{2I} = \frac{(98.1N)(4.00m)}{2(53.33kg \cdot m^2)} = 3.679rad/s^2$$

OR

$$w = mg ; I_{rod, end} = \frac{1}{3}mL^2$$

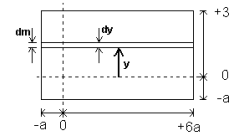
$$\sum \tau = w \frac{L}{2} = I\alpha$$

$$= \frac{mgL}{2} = \left(\frac{1}{3}mL^2\right)\alpha$$

$$\alpha = \frac{mgL}{2\left(\frac{1}{3}mL^2\right)} = \frac{g}{\frac{2}{3}L} = \frac{3g}{2L}$$

$$= \frac{3(9.81m/s^2)}{2(4.00m)} = 3.679rad/s^2$$

**One Fish, Two Fish, Red Fish, Star Fish! IN HONOR OF DR. SEUSS CENTENNIAL (50,000 points)**



2.) ☆(A) A plate of mass  $m$  has sides of  $4a$  and  $7a$ . Find the center of mass coordinates  $y_{cm}$  by integrating  $y_{cm} = \frac{1}{M} \int y dm$ , using the  $x$ - and  $y$ -axes as shown.

Guess center of mass is in the middle,  $y_{cm} = +a$

$$\lambda = \frac{M}{4a} ; dm = \lambda dy$$

$$y_{cm} = \frac{1}{M} \int y dm$$

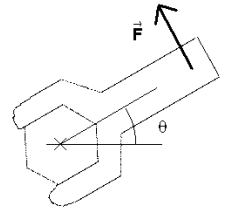
$$y_{cm} = \frac{1}{M} \int_{-a}^{+3a} y \lambda dy = \frac{\lambda}{M} \int_{-a}^{+3a} y dy$$

$$= \frac{\lambda}{M} \frac{y^2}{2} \Big|_{-a}^{+3a} = \frac{\frac{M}{4a}}{M} \left(\frac{1}{2}\right) \left((3a)^2 - (-a)^2\right) = \left(\frac{1}{8a}\right) (9a^2 - a^2)$$

$$= \frac{8a^2}{8a} = +a$$

☆(c) A torque  $\vec{\tau}$  to loosen a bolt consists of a force being applied at a distance of 30.0 cm (0.300 m) from the axis of rotation. As the bolt gets looser, it gets easier and easier to turn the bolt, so the force as a function of angle is given by  $F = \frac{C}{\theta}$ , where  $C$  is some constant with

appropriate units. If the total work done by applying this torque through two complete revolutions ( $\theta = 2\pi$  radians to  $6\pi$  radians) is 1500. J, then find  $C$ .



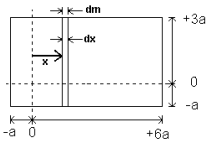
$$\tau = Fd = \frac{C}{\theta} R ; W = \int \tau d\theta = \int_{2\pi}^{6\pi} \frac{C}{\theta} R d\theta$$

$$= CR \ln \theta \Big|_{2\pi}^{6\pi} = CR(\ln(6\pi) - \ln(2\pi))$$

$$= CR \ln\left(\frac{6\pi}{2\pi}\right) = CR \ln 3 = 1500. J$$

$$C = \frac{1500. J}{(0.300m) \ln 3} = 4551 J/m = 4551 N \cdot rad$$

*Those quasi-units are slippery!*



☆(b) A plate of mass  $m$  has sides of  $4a$  and  $7a$ . Find the moment of inertia  $I$  of the plate about the  $y$ -axis as shown, by integrating  $I = \int r^2 dm$ .

*This is the corrected version of the problem.*

$$r \rightarrow x ; \quad \lambda = \frac{M}{7a} ; \quad dm = \lambda dx$$

$$I = \int r^2 dm = \int_{-a}^{+6a} x^2 \lambda dx = \lambda \int_{-a}^{+6a} x^2 dx = \lambda \frac{x^3}{3} \Big|_{-a}^{+6a}$$

$$= \frac{\lambda}{3} ((+6a)^3 - (-a)^3) = \frac{M}{7a} (216a^3 + a^3) = \frac{M}{21a} (217a^3) = \frac{31}{3} Ma^2$$

Although not necessary, we could check this result by using the Parallel Axis Theorem and the moment of inertia about the center of mass. The new axis is  $2.5a = 5L/14$  from the c.m.

$$a = \frac{L}{7} \quad \text{so} \quad I = \frac{31}{3} M \left( \frac{L}{7} \right)^2 = \frac{31}{147} ML^2$$

$$I_{PAT} = I_{cm} + MD^2 = \frac{1}{12} ML^2 + M \left( \frac{5L}{14} \right)^2 = \frac{1}{12} ML^2 + \frac{25}{196} ML^2$$

$$= \left( \frac{49}{588} + \frac{75}{588} \right) ML^2 = \frac{124}{588} ML^2 = \frac{31}{147} ML^2$$

(d) The wreck of the *RMS Titanic* lies 3821 meters below the surface of the ocean. Find the pressure at the bottom due to the sea water,  $\rho = 1030 \text{ kg/m}^3$ .

$$P = \rho gh = (1030 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(3821 \text{ m})$$

$$= 38,610,000 \text{ Pa}$$

☆(e) An object has a rotational motion that follows the following equations. Find the vector acceleration  $\vec{\alpha}$  at time  $t = 0$ .

$$\theta(t) = 1.00 \text{ rad} + (2.00 \text{ rad/s}^2) t^2 + (4.00 \text{ rad/s}^3) t^3$$

$$\theta(t) = 1.00 \text{ rad} + (2.00 \text{ rad/s}^2) t^2 + (4.00 \text{ rad/s}^3) t^3$$

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt} [1.00 \text{ rad} + (2.00 \text{ rad/s}^2) t^2 + (4.00 \text{ rad/s}^3) t^3]$$

$$= [0 + 2(2.00 \text{ rad/s}^2) t + 3(4.00 \text{ rad/s}^3) t^2]$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt}$$

$$\alpha = \frac{d}{dt} [2(2.00 \text{ rad/s}^2) t + 3(4.00 \text{ rad/s}^3) t^2]$$

$$= 2(2.00 \text{ rad/s}^2) + 2 \cdot 3(4.00 \text{ rad/s}^3) t$$

$$= (4.00 \text{ rad/s}^2) + (24.00 \text{ rad/s}^3) t$$

$$\alpha|_{t=0} = (4.00 \text{ rad/s}^2) + (24.00 \text{ rad/s}^3)(0)$$

$$= +4.00 \text{ rad/s}^2$$