XF.14a **205H**

PHYS-205(H14) (Kaldon-15032) Name S O L U T I O N

WMU-Spring 2004

Final Exam - 200,000 points + 40,000 ☆ points Check-Out: ☐ Q ☐ X ☐ T ☐ HNRS-290___

04/18/2004•Rev.8a

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions ☐☆2a ☐☆2b ☐☆2c ☐☆2e

FINAL EXAM [FORM - A]

PHYS-205H (KALDON-14)

SPRING 2004

WMU

CSI : Miami in Helsinki, Finland (!)

As The World Turns (50,000 points)

1.) (a) The Earth has a mass of 5.98×10^{24} kg and an average radius of 6,370,000 meters. Find the average mass-to-volume ratio of the Earth.

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi(6{,}370{,}000m)^3 = 1.083 \times 10^{21} m^3$$
$$\rho = \frac{m}{V} = \frac{5.98 \times 10^{24} kg}{1.083 \times 10^{21} m^3} = 5522 kg/m^3$$

(b) Find the moment of inertia, I, of the Earth. *Assume it is a uniform solid.*

$$I_{solid\ sphere} = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(5.98 \times 10^{24} kg)(6{,}370{,}000m)^2 = 9.706 \times 10^{37} kg \cdot m^2$$

(c) Find the period (T), frequency (f) and the angular velocity (ω) in SI units of the Earth as it spins on its axis.

$$T = 24.0 hours(3600\sec/hour) = 86{,}400\sec$$
$$f = \frac{1}{T} = \frac{1}{86{,}400\sec} = 0.00001157 Hz$$
$$\omega = 2\pi f = 2\pi(0.00001157 Hz) = 0.00007270 rad/\sec$$

(d) Find the angular momentum (L) and kinetic energy (K_{rot}) of the Earth as it spins on its axis.

$$L = I\omega = (9.706 \times 10^{37} kg \cdot m^2)(0.00007270 rad/\sec) = 7.056 \times 10^{37} kg \cdot m^2/\sec$$
$$K_{rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(9.706 \times 10^{37} kg \cdot m^2)(0.00007270 rad/\sec)^2 = 2.565 \times 10^{29} J$$

(e) The Earth is located an average distance of 1.496×10^{11} meters from the Sun, which has a mass of 1.991×10^{30} kg. Starting from Newton's Law of Universal Gravity, find the tangential speed, v, of the Earth as it goes in Uniform Circular Motion in orbit around the Sun. $G = 6.67\times10^{-11}\ N{\cdot}m^2/kg^2$

$$F_G = \frac{GmM}{r^2} = ma_c = \frac{mv^2}{r}$$

$$\frac{GM}{r^2} = \frac{v^2}{r} \ ; \ v^2 = \frac{GMr}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\,N\cdot m^2/kg^2)(1.991\times10^{30}\,kg)}{(1.496\times10^{11}\,m)}} = 29{,}790\,m/s$$

Survivor: All-Star Points (50,000 points)

2.) ☆(a) For the following $x(t)$ find the acceleration as a function of time. *All other constants of integration are zero.* $x(t) = 1.00m + (2.00m/s)\,t + \tfrac{1}{2}(4.00m/s^2)\,t^2$

$$x(t) = 1.00m + (2.00m/s)\,t + \tfrac{1}{2}(4.00m/s^2)\,t^2$$

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(1.00m + (2.00m/s)\,t + \tfrac{1}{2}(4.00m/s^2)\,t^2\right)$$

$$= 0 + (2.00m/s) + \tfrac{1}{2}(2)(4.00m/s^2)\,t = (4.00m/s^2)\,t$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = \frac{d}{dt}\left((4.00m/s^2)\,t\right) = 4.00m/s^2$$

☆(b) For $\alpha(t) = 4.00rad/s^2$, find the angle as a function of time. *All other constants of integration are zero.*

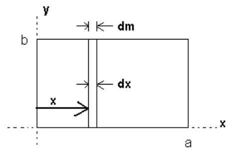
$$\omega = \int \alpha\,dt = \int (4.00rad/s^2)\,dt = (4.00rad/s^2)\,t + C$$

$$= (4.00rad/s^2)\,t$$

$$\theta = \int \omega\,dt = \int (4.00rad/s^2)\,t\,dt = \tfrac{1}{2}(4.00rad/s^2)\,t^2 + C$$

$$= (2.00rad/s^2)\,t^2$$

☆(c) A plate of mass m = 5.00 kg has dimensions a = 1.00 m and b = 0.480 m. Find the center of mass coordinates x_{cm} by integrating $x_{cm} = \frac{1}{M}\int x\,dm$. The origin is located at the lower left corner.

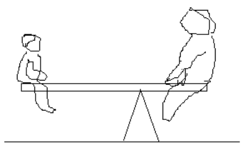


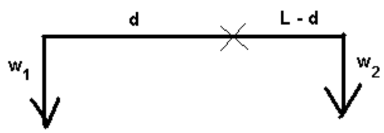
$$\lambda = \frac{M}{L} = \frac{5.00kg}{1.00m} = \frac{m}{a} \ ; \ dm = \lambda\,dx$$

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^a x\,\lambda\,dx = \frac{\lambda}{M}\int_0^a x\,dx$$

$$= \frac{\lambda}{M}\left(\frac{x^2}{2}\right)\Bigg|_0^a = \frac{m/a}{m}\left(\frac{a^2}{2} - 0\right) = \frac{a}{2} = \frac{1.00m}{2} = 0.500m$$

(d) A 3.00 meter long teeter-totter has a 25.0 kg kid on the left end and a 40.0 kg kid on the right end. The pivot is located a distance d from the left end. Find d. *Ignore the weight of the teeter-totter itself.*





$$\begin{aligned}
&\sum \tau = m_1 g d - m_2 g(L-d) = 0\\
&m_1 g d = m_2 g(L-d)\\
&m_1 d = m_2 L - m_2 d\\
&m_1 d + m_2 d = m_2 L\\
&(m_1+m_2)d = m_2 L\\
&d = \frac{m_2 L}{(m_1+m_2)} = \frac{(40.0kg)(3.00m)}{(25.0kg+40.0kg)} = 1.846m
\end{aligned}$$

☆(e) A mass on a spring has its position as a function of time given as $x(t) = A\cos(\omega t) + B\sin(\omega t)$. Find the speed v and acceleration a at time $t = 0$.

$$\begin{aligned}
&x(t) = A\cos(\omega t) + B\sin(\omega t)\\
&v = \frac{dx}{dt} = \frac{d}{dt}\left(A\cos(\omega t) + B\sin(\omega t)\right) = -\omega A\sin(\omega t) + \omega B\cos(\omega t)\\
&v(0) = -\omega A\sin(0) + \omega B\cos(0) = 0 + \omega B = \omega B\\
&a = \frac{dv}{dt} = \frac{d}{dt}\left(-\omega A\sin(\omega t) + \omega B\cos(\omega t)\right) = -\omega^2 A\cos(\omega t) - \omega^2 B\sin(\omega t)\\
&a(0) = -\omega^2 A\cos(0) - \omega^2 B\sin(0) = -\omega^2 A - 0 = -\omega^2 A
\end{aligned}$$

CSI: Crime Scene Investigators (50,000 points)

3.) On April 8th, 2004, CBS had a rerun of a *CSI* episode. The "B-story" was about an idiot target shooting a 9mm automatic pistol in his backyard. He was "distracted" when the neighbor next door yelled at him to stop – and one shot went high. A woman several blocks away was killed when the wayward bullet penetrated 4". Forensic analysis leads them to determine that a 9mm bullet penetrating 4" is traveling at only 550 ft/sec (168 m/s), while a full-load shot at 1100 ft/sec (336 m/s) will penetrate 12". *"How far does a 9mm bullet have to travel to slow down from 1100 ft/sec to 550 ft/sec?" "1800 feet."* (a) Find the constant acceleration a that will slow a bullet from 336 m/s to 168 m/s in 1800 feet (549 meters).†

$$\begin{aligned}
&v^2 = v_0^2 + 2a(x-x_0)\ ;\ \ x_0 = 0\\
&v^2 = v_0^2 + 2ax\\
&2ax = v^2 - v_0^2\\
&a = \frac{v^2 - v_0^2}{2ax} = \frac{(168m/s)^2 - (336m/s)^2}{2(549m)}\\
&\ \ = -77.11m/s^2
\end{aligned}$$

"Can a 9mm bullet even travel 1800 feet? That's three football fields." "600 feet to drop to ground if fired horizontally. But it could have been fired at an angle." (b) Find the time t it takes for a bullet traveling at an average speed of 252 m/s to fall from a horizontal gun held 1.50 meters above the ground.

$$\begin{aligned}
&y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2\ ;\ \ y = 0,\ v_{0y} = 0\\
&0 = y_0 - \tfrac{1}{2}gt^2\\
&y_0 = \tfrac{1}{2}gt^2\\
&t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(1.50m)}{9.81m/s^2}} = 0.5530\sec
\end{aligned}$$

(c) Find the horizontal distance d traveled for a bullet traveling at an average horizontal speed of 252 m/s for the time t from part (b). *If you didn't get a time in (b), use t = 1.50 seconds.*

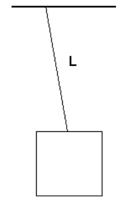
$$d = v\,t = (252m/s)(0.5530\sec) = 139.4m$$

(d) Modern electronics allow one to measure the muzzle speed of a bullet as it leaves the barrel of a gun. In the "old days", however, one used a ballistic pendulum – basically a block of wood hanging by a cable.

† Actually, the bullet would be slowed by the high-speed drag force, F = -cv², but you do not want to work this problem with a variable, velocity dependent force. (grin)

Suppose a 100. kg cube-shaped block of wood ($\rho = 650.\ kg/m^3$) is suspended from the ceiling by a cable 1.75 meters long. Find the period T of this simple pendulum if it is displaced slightly from rest (no firearms involved).

$$\omega = \sqrt{\frac{g}{L}}\ ;\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$
$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.75m}{9.81m/s^2}} = 2.654\sec$$



But wait – the length of the cable is to the center-of-mass of the bob… so a better answer is:

$$\rho = \frac{m}{V}\ ;\ V = \frac{m}{\rho} = \frac{100.kg}{650.kg/m^3} = 0.1538m^3$$
$$V = d^3\ ;\ d = \sqrt[3]{V} = \sqrt[3]{0.1538m^3} = 0.5358m$$
$$L = L_0 + \tfrac{1}{2}d = 1.75m + \tfrac{1}{2}(0.5358m) = 2.018m$$
$$\omega = \sqrt{\frac{g}{L}}\ ;\ f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$
$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.018m}{9.81m/s^2}} = 2.850\sec$$

(e) The recoil of the pistol when it is fired can be described by two different, but important Physics concepts. 5000 points for the Law and 5000 points for the Conservation principle. *Short answers*.

Newton's Third Law & Conservation of Linear Momentum

(recoil is a totally inelastic collision run backwards in time)

The Dr. Phil Show (50,000 points)

4.) (a) A 2000 Lamborghini Diablo (mass = 1625 kg) generates 550. hp at full power. How much work does the engine do in 4.00 seconds? *1 hp = 746 W.*

$$(550.hp)(746W/hp) = 410{,}300W$$
$$P = \frac{W}{t}$$
$$W = Pt = (410{,}300W)(4.00\sec) = 1{,}641{,}000J$$

(b) The actual efficiency of the hot engine at full power is 54.0% (0.540). For the work done in (a), how much energy is wasted? *If you didn't get an answer to (a), use W = 162,000 J.*

$$\varepsilon_{actual} = \frac{W}{Q_H}\ ;\ Q_H = \frac{W}{\varepsilon_{actual}} = \frac{1{,}641{,}000J}{0.540} = 3{,}039{,}000J$$
$$Q_H = W + Q_C$$
$$Q_C = Q_H - W = 3{,}039{,}000J - 1{,}641{,}000J = 1{,}398{,}000J$$

(c) The operating temperatures of the V-12 engine of the Diablo are 105°C (221°F) and 866 K (1100°F). Find the 2nd Law Efficiency of this engine.

$$T_C = 105°C = 378K$$
$$\varepsilon_{Carnot} = 1 - \frac{T_C}{T_H} = 1 - \frac{378K}{866K} = 0.5635$$
$$\varepsilon_{2nd\ Law} = \frac{\varepsilon_{actual}}{\varepsilon_{Carnot}} = \frac{0.540}{0.5635} = 0.9583 = 95.83\%$$

(d) Starting from rest, how fast will the Diablo be going after 4.00 seconds of full power? Assume no slipping, no non-conservative forces.

$$W = \Delta K = K_f - K_i\ ;\ K_f = W = \tfrac{1}{2}mv^2$$
$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(1{,}641{,}000J)}{1625kg}} = 44.95m/s\ \ (106mph!)$$

(e) For a constant speed, power can be written as $P = F \cdot v$. If the top speed of the Diablo is 225 m.p.h. (101 m/s), and is limited by the maximum power and high speed air drag of the form $F_{drag} = -C\,v^2$, find C.

$$P = Fv \quad ; \quad F = -Cv^2$$

$$P = \left(Cv^2\right)v = Cv^3$$

$$C = \frac{P}{v^3} = \frac{410{,}300W}{\left(101m/s\right)^3} = 0.3982W \cdot s^3 / m^3 = 0.3982kg/m$$

$$\left(W \cdot s^3 / m^3\right) = \left(\frac{J \cdot s^3}{s \cdot m^3}\right) = \left(\frac{N \cdot m \cdot s^3}{s \cdot m^3}\right) = \left(\frac{kg \cdot m \cdot m \cdot s^3}{s^2 \cdot s \cdot m^3}\right) = \left(\frac{kg}{m}\right)$$