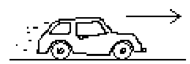
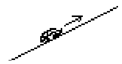
Wheels (50,000 points)



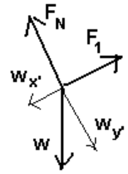
1.) (a) Jenny hops into her economy car (*m = 1110 kg*) and takes off as fast as she can, $a = 3.28\ m/s^2$. Find this maximum force F_1 that is moving the car.

$$F_1 = ma = (1110kg)(3.28m/s^2) = 3641N$$

(b) If Jenny is driving up a hill at a constant speed of 25.0 m/s, find the maximum angle θ of the hill given the maximum force F_1. *If you didn't get an answer to (a), use F_1 = 5550 N*.

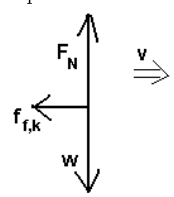


$$\sum F_{x'} = F_1 - w_{x'} = 0$$
$$F_1 = w_{x'} = mg\sin\theta$$
$$\sin\theta = \frac{F_1}{mg} = \frac{3641N}{(1110kg)(9.81m/s^2)} = 0.3344$$
$$\theta = \sin^{-1}[0.3344] = 19.5°$$



(c) Jenny is driving on a flat, level road at a speed of 25.0 m/s when she sees that there's construction ahead and all the cars have stopped. If she slams on the brakes, she will skid to a stop. If the coefficients of friction are 0.950 and 0.680, find the distance d that it would take for Jenny to come to a stop.

$$\sum F_y = F_n - mg = 0$$
$$F_n = mg$$
$$F_{f,k} = \mu_k F_n = \mu_k mg$$
$$\sum F_x = -F_{f,k} = ma_x$$



by kinematic equations

$$a_x = \frac{-\mu_k mg}{m} = -\mu_k g$$
$$= -(0.680)(9.81m/s^2)$$
$$= -6.671m/s^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$0 = v_0^2 + 2ad$$
$$2ad = -v_0^2$$
$$d = -\frac{v_0^2}{2a} = -\frac{(25.0m/s)^2}{2(-6.671m/s^2)}$$
$$= 46.84m$$

OR

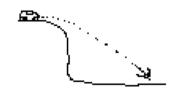
by Work-Energy Theorem

$$W = Fd = -F_{f,k}d = -\mu_k mgd$$
$$W = \Delta K = K_f - K_i = -K_i$$
$$-\mu_k mgd = -K_i$$
$$\mu_k mgd = \tfrac{1}{2}mv_0^2$$
$$\mu_k gd = \tfrac{1}{2}v_0^2$$
$$d = -\frac{v_0^2}{2\mu_k g}$$
$$= -\frac{(25.0m/s)^2}{2(0.680)(-9.81m/s^2)}$$
$$= 46.85m$$

(d) Unfortunately, before Jenny comes to a stop there's a big sedan (*m = 2270 kg*) which has slowed down in front of her. If she hits it totally inelastically, when her car was traveling at $v_1 = 12.0\ m/s$ and the sedan was moving at $v_2 = 8.00\ m/s$, find the speed V of the wreck.

$$p_{before} = p_{after}$$
$$m_1v_1 + m_2v_2 = (m_1 + m_2)V$$
$$V = \frac{m_1v_1 + m_2v_2}{(m_1 + m_2)} = \frac{(1110kg)(12.0m/s) + (2270kg)(8.00m/s)}{(1110kg + 2270kg)}$$
$$= 9.314m/s$$

(e) Instead, Jenny swerves at $v_i = 12.0\ m/s$ only to find herself sailing off of a cliff and crashing into the valley 166 meters below. Use Conservation of Energy to find her final speed v_f just as she hits the ground.



$$mgh_1 + \tfrac{1}{2}mv_1^2 = mgh_2 + \tfrac{1}{2}mv_2^2 \quad ; \quad h_2 = 0$$
$$mgh_1 + \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_2^2$$
$$gh_1 + \tfrac{1}{2}v_1^2 = \tfrac{1}{2}v_2^2$$
$$v_2^2 = 2gh_1 + v_1^2$$
$$v_2 = \sqrt{2gh_1 + v_1^2}$$
$$= \sqrt{2(9.81m/s^2)(166m) + (12.0m/s)^2}$$
$$= 58.32m/s$$

What's the Point of Star Problems? ANSWER: (50,000 points)

2.) The *sprangle* force is given as $F = -C x^5$. If the *sprangle* force is a conservative force, then calculate the equation of the potential energy term $U_{sprangle}$.

$$U = -W$$
$$W = \int \vec{F} \cdot d\vec{s} = \int (-Cx^5\,\hat{i}) \cdot dx\,\hat{i} = -C\int x^5 dx$$
$$= -C\frac{x^6}{6} + const. \quad ; \; declare\; const. = 0$$
$$U = -W = -\left(-C\frac{x^6}{6}\right) = C\frac{x^6}{6}$$

As with U_g and U_s , we can set $U = 0$ wherever we like.

☆(b) An object of mass $m = 9.00\ kg$ begins its motion at $x_0 = 9.00\ m$, $v_0 = 9.00\ m/s$, $a_0 = 9.00\ m/s^2$ and an initial jerk of $j_0 = 9.00\ m/s^3$. Find the equation for the force, F, acting on this object, where the motion of the object is determined by the following equation:

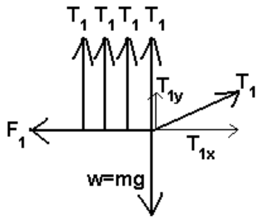
$$\frac{d^4x}{dt^4} = 9.00\ m/s^4$$

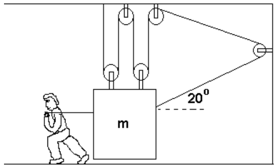
$$a_x = \frac{d^2x}{dt^2} \quad ; \quad \frac{d^4x}{dt^4} = 9.00\ m/s^4$$
$$\frac{d^3x}{dt^3} = \int \frac{d^4x}{dt^4}\, dt = \int 9.00\ m/s^4\, dt$$
$$= (9.00\ m/s^4)\,t + C \quad ; \; (C = j_0 = 9.00\ m/s^3)$$
$$= (9.00\ m/s^4)\,t + 9.00\ m/s^3$$
$$\frac{d^2x}{dt^2} = \int \frac{d^3x}{dt^3}\, dt = \int \left((9.00\ m/s^4)\,t + 9.00\ m/s^3\right) dt$$
$$= (9.00\ m/s^4)\frac{t^2}{2} + (9.00\ m/s^3)\,t + C \quad ; \; (C = a_0 = 9.00\ m/s^2)$$
$$a_x = (4.50\ m/s^4)\,t^2 + (9.00\ m/s^3)\,t + 9.00\ m/s^2$$
$$F = ma = (9.00kg)\left[(4.500\ m/s^4)\,t^2 + (9.00\ m/s^3)\,t + 9.00\ m/s^2\right]$$
$$= (40.50\ N/s^2)\,t^2 + (81.00\ N/s)\,t + 81.00\ N$$

☆(c) Find the work done when $\vec{F} = 3.00\,{}^{N}\!/_{m}\; x\,\hat{i} + 4.00\,{}^{N}\!/_{m^2}\; y^2\,\hat{j}$ and the displacement is $\vec{s} = 5.00$ m @ 30°.

$$\vec{s} = 5.00m\,@\,30°$$
$$x = (5.00m)(\cos 30°) = 4.330m$$
$$y = (5.00m)(\sin 30°) = 2.500m$$
$$\vec{F} = 3.00\,{}^{N}\!/_{m}\; x\,\hat{i} + 4.00\,{}^{N}\!/_{m^2}\; y^2\,\hat{j}$$
$$W = \int \vec{F} \cdot d\vec{s} = \int_0^x F_x dx + \int_0^y F_y dy$$
$$= \int_0^x 3.00\,{}^{N}\!/_{m}\; x\,dx + \int_0^y 4.00\,{}^{N}\!/_{m^2}\; y^2 dy$$
$$= (3.00\,{}^{N}\!/_{m})\int_0^x x\,dx + (4.00\,{}^{N}\!/_{m^2})\int_0^y y^2 dy$$
$$= (3.00\,{}^{N}\!/_{m})\frac{x^2}{2}\Bigg|_0^{4.330m} + (4.00\,{}^{N}\!/_{m^2})\frac{y^3}{3}\Bigg|_0^{2.500m}$$
$$= \frac{(3.00\,{}^{N}\!/_{m})(4.330m)^2}{2} + \frac{(4.00\,{}^{N}\!/_{m^2})(2.500m)^3}{3}$$
$$= 28.12J + 20.83J = 48.95J$$

(d) A block of mass *m* is attached to several pulleys, plus a rope being pulled by Fred, but the block is not moving. Sketch the Free Body Diagram for the block *m* and then write the accompanying sum of forces equations in the *x*- and *y*-directions. *But do not solve the equations!*





$$\begin{aligned}
\sum F_y &= T_1 + T_1 + T_1 + T_1 + T_{1y} - mg = 0 \\
&= 4T_1 + T_1 \sin 20° - mg = 0 \\
&= T_1(4 + \sin 20°) - mg = 0 \\
&= T_1(4.342) - mg = 0 \\
\sum F_x &= T_{1x} - F_1 = 0 \\
&= T_1 \cos 20° - F_1 = 0 \\
&= 0.9397 T_1 - F_1 = 0
\end{aligned}$$