X3.11A **2070**

PHYS-2070(11) (Kaldon-45451) Name S O L U T I O N

WMU - Fall 2008

Exam 3A - 100,000 points + 20,000 ☆ points

11/24/2008•Rev.5.r1

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions ☐☆2a ☐☆2b ☐☆2c ☐☆2e

EXAM 3 [FORM - A]
PHYS-2070 (KALDON-11)
FALL 2008
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$$k = 8.988\times10^{9}\ N\cdot m^2/C^2$$

$$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$$

$$\mu_0 = 4\pi\times10^{-7}\ T\cdot m/A$$

$$e = 1.602\times10^{-19}\,C$$

$$m_e = 9.11\times10^{-31}\,kg$$

Loopy Hot Physics (50,000 points)

1.) (a) A hair dryer has a heating element which is both an inductor and a resistor. Hooked up to an A.C. power source with V_{rms} = *121 volts*, *62.0 Hz*, the rms power of the heating element is 1080 W. Find the resistance R of this heating element. *Assume that this device is purely resistive for the moment.*

$$P_{rms} = \frac{V_{rms}^2}{R}$$

$$R = \frac{V_{rms}^2}{P_{rms}} = \frac{(121 volts)^2}{1080W} = 13.56\Omega$$

(b) The heating element is 0.500 cm in diameter and consists of 200. turns of high resistance wire along a length of 5.00 cm. Find the inductance L of this heating element.

$$L = \frac{\mu_0 N^2 A}{l} \quad ; \quad A = \pi r^2 = \pi D^2/4$$

$$= \frac{\left(4\pi\times10^{-7}\ \tfrac{T\cdot m}{A}\right)(200.)^2\left(\pi(0.00500m)^2/4\right)}{(0.0500m)}$$

$$= 0.00001974H = 1.974\times10^{-5}H$$

(c) To minimize the impedance Z of this hair dryer, what capacitor C must be hooked up in series to the heating element?

$$\omega = 2\pi f = 2\pi(62.0Hz) = 389.6 rad/\sec$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad ; Z = \min\ when\ X_L - X_C = 0$$

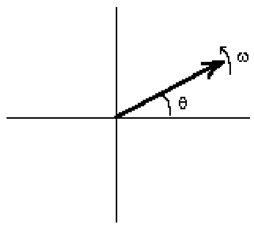
$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C} \quad ; \left(X_L = \omega L = (389.6 rad/\sec)(1.974\times10^{-5}H) = 0.007691\Omega\right)$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{(389.6 rad/\sec)^2(1.974\times10^{-5}H)} = 0.3337F$$

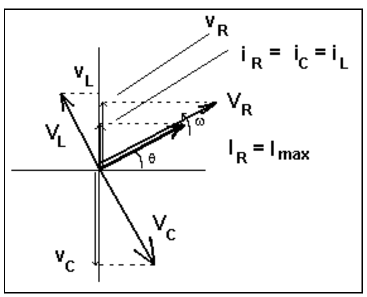
(d) Use the inductance L to find the maximum magnetic flux, Φ_B, inside the coil of wire. *If you didn't get an answer to (b), use L = 0.125 H.*

$$V_{\max} = \frac{V_{rms}}{0.7071} = \frac{121 volts}{0.7071} = 171.1 volts$$
$$V_{\max} = I_{\max} Z = I_{\max} R$$
$$I_{\max} = \frac{V_{\max}}{R} = \frac{171.1 volts}{13.56\Omega} = 12.62\,A$$
$$L = \frac{N\Phi_B}{I}$$
$$\Phi_B = \frac{LI}{N} = \frac{(1.974\times10^{-5}H)(12.62\,A)}{(200.)}$$
$$= 1.246\times10^{-6}\,T\cdot m^2$$



(e) For the LRC circuit in (d), the vector in the phasor diagram at the right represents V_R, sketch in I_{max}, V_L , V_C and show the relevant instantaneous components, with labels.

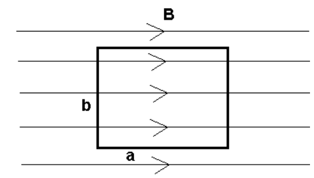
Not to scale.



And Now The Big Name Stars Of Our Exam! (50,000 points)

2.) Ampère's Law says that $\oint \vec{B}\cdot d\vec{s} = \mu_0 I$. Construct suitable Ampèrean Loops[1] in the *xy* plane to evaluate the integral and show that Ampère's Law is true for the following two cases: ☆(a) a region of space with a uniform $\vec{B}$ -field in the -*x* direction…

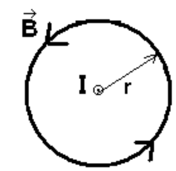
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$
$$\int_L \vec{B}\cdot d\vec{s} + \int_{top} \vec{B}\cdot d\vec{s} + \int_R \vec{B}\cdot d\vec{s} + \int_{bottom} \vec{B}\cdot d\vec{s} = \mu_0(0)$$
$$0 + \int_{top} B\,ds + 0 + \int_{bottom} B ds = \mu_0(0)$$
$$B\int_0^a ds - B\int_0^a ds = \mu_0(0) \quad ; alt\ 2nd\ term\ + B\int_a^0 ds$$
$$B\,a - Ba = \mu_0(0)$$
$$0 = 0$$



Ampere's Law is true because there is no current enclosed.

☆(b) … and a region of space with a wire having a current *I* in the +*z* direction.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enclosed} \quad ; \quad ds = r\,d\theta \ ; d\vec{s} = ds\,\hat{\theta} \ ; \vec{B} = B\ \hat{\theta}$$
$$\int_0^{2\pi} B\,r\,d\theta = Br\int_0^{2\pi} d\theta = \mu_0\, I$$
$$Br\theta\Big|_0^{2\pi} = Br(2\pi - 0) = \mu_0\, I$$
$$B = \frac{\mu_0\, I}{2\pi\, r}$$



Which is, of course, the equation for B from a long, straight current carrying wire.

[1] Hint: What are the two easiest to calculate geometrical shapes you can come up with? One is a square, the other is a circle.

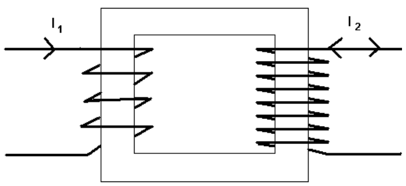
☆(c) Find the magnetic flux, Φ_B, in a circular area of radius a in the xy plane, where the magnetic field is $\vec{B} = b\,r\,\hat{k}$, where b is just some constant.

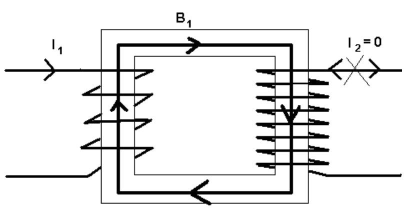
$$\Phi_B = \int \vec{B}\cdot d\vec{A}\ ;\ dA = r\,dr\,d\theta\ ;\ \vec{B} = b\,r\,\hat{k}\ ;\ d\vec{A} = dA\hat{k}$$

$$\Phi_B = \int B\,dA = \int_0^a \int_0^{2\pi} (br)r\,dr\,d\theta$$

$$= b\int_0^a \int_0^{2\pi} r^2\,dr\,d\theta = b\left.\frac{r^3}{3}\right|_0^a \left.\theta\right|_0^{2\pi} = \frac{2\pi\,b\,a^3}{3}$$

(d) A transformer, which has been running all morning, is shown below. If the steady primary current I_1 is $5.00\ A$ and going to the right as shown, then explain whether the induced current of the secondary, I_2, goes to the LEFT or to the RIGHT. You must sketch in the relevant magnetic field B_1 from the primary coil, and the induced magnetic field B_2 from the secondary coil. Finally, if $V_1 = 117\ Volts$, then find V_2.





The B-field from I_1 goes up from the center of the coil and then stays in the iron frame, going clockwise. The B-field goes DOWN through coil 2 and is CONSTANT, so that coil 2 has NO RESPONSE because there is NO CHANGE in its magnetic flux.

Because this is DC, $V_2 = 0$.

☆(e) Show whether the charge as a function of time, $q(t) = Q_0\,e^{-\frac{t}{\sqrt{LC}}}$, is a solution to the differential equation for an ideal LC circuit is $\frac{d^2q}{dt^2} = -\frac{1}{LC}\,q$.

$$q(t) = Q_0\,e^{-\frac{t}{\sqrt{LC}}}$$

$$\frac{dq}{dt} = \frac{d}{dt}\left[Q_0\,e^{-\frac{t}{\sqrt{LC}}}\right] = -\frac{Q_0}{\sqrt{LC}}\,e^{-\frac{t}{\sqrt{LC}}}$$

$$\frac{d^2q}{dt^2} = \frac{d}{dt}\left[\frac{dq}{dt}\right] = \frac{d}{dt}\left[-\frac{Q_0}{\sqrt{LC}}\,e^{-\frac{t}{\sqrt{LC}}}\right]$$

$$= -\left(-\frac{1}{\sqrt{LC}}\right)\frac{Q_0}{\sqrt{LC}}\,e^{-\frac{t}{\sqrt{LC}}} = +\frac{Q_0}{LC}\,e^{-\frac{t}{\sqrt{LC}}}$$

$$\frac{d^2q}{dt^2} = -\frac{1}{LC}\,q$$

$$+\frac{Q_0}{LC}\,e^{-\frac{t}{\sqrt{LC}}} = -\frac{1}{LC}\left(Q_0\,e^{-\frac{t}{\sqrt{LC}}}\right)$$

$$OR\ldots\quad +1 \neq -1$$

This is NOT a solution to the LC oscillator equation.