It's A Matter Of High Voltage Inductive Reasoning (50,000 points)

1.) A solenoid coil has 50,000 turns, a cross-sectional area of 0.001257 m² and a length *l = 12.0 cm = 0.120 m*. Find (a) the inductance *L* of the coil and the magnetic flux Φ_B passing through the coil if the current is *I = 1.25 A*.

$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi\times10^{-7}\ T\cdot m/A)(50{,}000)^2(0.001257m^2)}{(0.120m)} = 32.91H$$

$$L = \frac{N\Phi_B}{I}$$
$$\Phi_B = \frac{LI}{N} = \frac{(32.91H)(1.25A)}{(50{,}000)} = 0.0008228\,T\cdot m^2$$

(b) If this solenoid is placed in an AC circuit with V_{rms} *= 125 volts* at *61.0 Hz*, find the capacitor *C* which will minimize the impedance. *If you didn't get an answer to (a), use L = 25.0 H.*

$$\omega = 2\pi f = 2\pi(61.0Hz) = 383.3 rad/\sec$$
$$X_L = X_C$$
$$\omega L = \frac{1}{\omega C}$$
$$C = \frac{1}{\omega^2 L} = \frac{1}{(383.3 rad/\sec)^2(32.91H)} = 2.068\times10^{-7}F = 0.2068\mu F = 206{,}800\,pF$$

(c) A house is connected to an A.C. power source with V_{rms} *= 125 volts*, I_{rms} *= 125 A, 61.0 Hz*. A transformer raises the voltage to V_{rms} *= 375,000 volts* for the high voltage power line. Find the I_{rms} current and the maximum current I_{max} in the power line.

$$P = I_1V_1 = I_2V_2$$
$$I_1 = \frac{I_2V_2}{V_1} = \frac{(125A)(125V)}{(375{,}000V)} = 0.04167A$$

$$I_{rms} = 0.7071 I_{\max}$$
$$I_{\max} = \frac{I_{rms}}{0.7071} = \frac{0.04167A}{0.7071} = 0.05893A$$

(d) At a particular time, the current in the overhead power lines is I_{max} in the *+x* direction for wire 1 and I_{max} in the *-x* direction for wire 2. If the two wires are 10.0 meters apart and 125 meters long, find the magnitude of the maximum magnetic force of wire 1 on wire 2. *Assume the wires are horizontal and straight. If you did not get an answer to (c), use* I_{max} *= 125 A.*

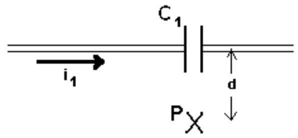
$$F_{1on2} = \frac{\mu_0 I_1 I_2 l_2}{2\pi d} = \frac{(4\pi\times10^{-7}\ T\cdot m/A)(0.05893A)(0.05893A)(125m)}{2\pi(10.0m)} = 8.682\times10^{-9}N$$

(e) At the same time, with the current in the overhead power lines is I_{max} in the *+x* direction for wire 1, find the maximum magnetic field B_1 at a point *P* 25.0 meters directly below.

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi\times10^{-7}\ T\cdot m/A)(0.05893A)}{2\pi(25.0m)} = 4.714\times10^{-10}T$$

And The Star Problems Came Out In The Afternoon… (50,000 points)

2.) ☆(a) A point *P* is located a distance *d* below a capacitor C_1. If the electric flux between the plates of the capacitor is $\Phi_E = (387 V\cdot m/\sec)\, t$, find the current i_1 which will give the same magnetic field *B* at the point *P* that the capacitor does, using the Ampere-Maxwell equation.



$$\Phi_E = (387 V\cdot m/\sec)\, t$$
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \varepsilon_0\mu_0 \frac{d\,\Phi_E}{d\,t}$$
$$\mu_0 I = \varepsilon_0\mu_0 \frac{d\,\Phi_E}{dt}$$
$$I = \varepsilon_0 \frac{d\,\Phi_E}{dt}$$
$$= \varepsilon_0 \frac{d}{dt}\left[(387 V\cdot m/\sec)\, t\right]$$

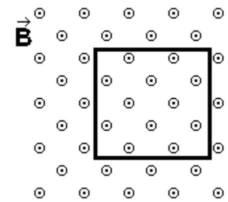
$$I = \varepsilon_0 (387 V\cdot m/\sec)$$
$$= \left(8.85\times10^{-12}\, C^2/_{N\cdot m^2}\right)(387 V\cdot m/\sec)$$
$$= 3.425\times10^{-9} A$$

☆(b) The current into an inductor, is given as $I(t) = 1.00A + (2.00A/\sec^2)t^2 - (4.00A/\sec^4)t^4$. Use the equation for self-inductance $\mathcal{E}_L = -L\frac{dI}{dt}$ to find the induced *emf* at the time *t = 2.00 sec* if *L = 0.155 H*.

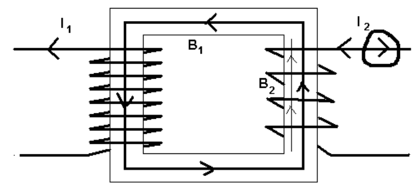
$$I(t) = 1.00A + (2.00A/\sec^2)t^2 - (4.00A/\sec^4)t^4$$
$$\mathcal{E}_L = -L\frac{dI}{dt} = -L\frac{d}{dt}\left[1.00A + (2.00A/\sec^2)t^2 - (4.00A/\sec^4)t^4\right]$$
$$= -L\left(0 + 2(2.00A/\sec^2)t - 4(4.00A/\sec^4)t^3\right)$$
$$= L\left(-(4.00A/\sec^2)t + (16.0A/\sec^4)t^3\right)$$
$$= (0.155H)\left((-4.00A/\sec^2)(2.00\sec) + (16.0A/\sec^4)(2.00\sec)^3\right)$$
$$= 18.60 volts$$

☆(c) Consider a square in the *xy*-plane with sides of length *a* and a magnetic field, $\vec{B} = b\,x\,y\,\hat{k}$, where *b* is just some constant. Find the magnetic flux $\Phi_B = \int \vec{B}\cdot d\vec{A}$. *The lower left corner of the square is at the origin, x=y=0.*

$$\Phi_B = \int \vec{B}\cdot d\vec{A}\ ;\ d\vec{A} = dx\,dy\ \hat{k};\ \vec{B} = b\,xy\,\hat{k}$$
$$\Phi_B = \int B\,dA = \int_0^a\int_0^a b\,xy\,dxdy$$
$$= b\int_0^a x\,dx\int_0^a y\,dy = b\left.\frac{x^2}{2}\right|_0^a \left.\frac{y^2}{2}\right|_0^a = \frac{ba^2\,a^2}{4}$$
$$= \frac{ba^4}{4}$$



(d) A transformer is shown below. If the primary current I_1 is *5.00 A* going to the left as shown and decreasing, then explain whether the induced current of the secondary, I_2, goes to the LEFT or to the RIGHT. You must sketch in the relevant magnetic field B_1 from the primary coil, and the induced magnetic field B_2 from the secondary coil. Finally, if V_1 = *117 Volts*, then find V_2.



$$V_2 = \frac{N_2}{N_1}V_1 = \frac{(4)}{(8)}(117 volts) = 58.50 volts$$

The B-field from I_1 goes down from the center of the coil and then stays in the iron frame, going counterclockwise. The B-field goes UP through coil 2 and is DECREASING, so that coil 2's RESPONSE is to try to maintain its magnetic flux.

☆(e) Starting from the equation for an inductor with *N* turns, $L = \frac{N\Phi_B}{I}$, show that the right-hand equality of the self-induced emf equation for a coil in a magnetic field $\mathcal{E}_L = -N\frac{d\,\Phi_B}{d\,t} = -L\frac{d\,I}{d\,t}$, is true.

$$L = \frac{N\Phi_B}{I}$$
$$N\Phi_B = LI$$
$$\frac{d}{dt}[N\Phi_B] = \frac{d}{dt}[LI]$$
$$N\frac{d\Phi_B}{dt} = L\frac{d\,I}{dt}$$
$$\mathcal{E}_L = -N\frac{d\,\Phi_B}{d\,t} = -L\frac{d\,I}{d\,t}$$