PHYS-207 (8) (Kaldon-52328) Name **S O L U T I O N**

WMU - Spring 2003

Exam 2A - 100,000 points + 20,000 ☆ points Book Title ____________

02/25/2003•Rev.A6

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers

Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations

Feel Free to Ask Any Questions ☐☆2a ☐☆2b ☐☆2c ☐☆2e

EXAM 2 [FORM - A]
PHYS-207 (KALDON-8)

SPRING 2003

WMU

$k = 8.988 \times 10^{9}\ N \cdot m^2 / C^2$ $\quad$ $e = 1.602 \times 10^{-19}\ C$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / N \cdot m^2$ $\quad$ $m_e = 9.11 \times 10^{-31}\ kg$

Don't Panic. Just Follow the Wires and Make the Connections (50,000 points)

1.) (a) You are given two square metal plates with sides $a = 5.00\ cm$. Design an air filled capacitor, such that when hooked up to a 9.00 volt battery and fully charged, it will spark across the gap and discharge. What is the capacitance C of this badly designed capacitor? *Do not say that the answer is zero… (grin).*

$$E_{\max} = 3{,}000{,}000\ V/m$$

$$V = E_{\max}\, d$$

$$d = \frac{V}{E_{\max}} = \frac{9.00\,volts}{3{,}000{,}000\ V/m} = 0.000\,003\,00\,m$$

$$C = \varepsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12}\ C^2/N \cdot m^2\right) \frac{(0.0500\,m)^2}{(0.000\,003\,00\,m)}$$

$$= 7.375 \times 10^{-9}\,F = 7375\,pF$$

(b) A brand fresh 6.00 volt battery acts like a perfect battery at time $t = 0$ and delivers a current of 1.63 A. At some later time, as the real battery weakens, the current drops to 1.51 A. Find the internal resistance r of the weak battery.

Perfect Battery

$$\mathcal{E} = i\,R$$

$$R = \frac{\mathcal{E}}{i} = \frac{6.00\,volts}{1.63\,A} = 3.681\,\Omega$$

Real Battery = Perfect Battery w/ Internal Resistance

$$V = IR \quad or \quad \mathcal{E} = I(r + R)$$

$$\mathcal{E} - I\,r = I\,R$$

$$\mathcal{E} = I\,r + I\,R$$

$$I\,r = \mathcal{E} - I\,R$$

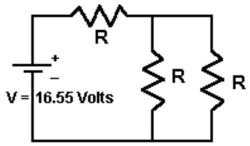
$$r = \frac{\mathcal{E} - I\,R}{I}$$

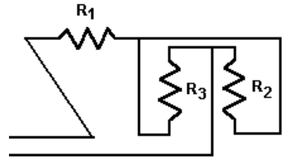
$$= \frac{6.00\,volts - (1.51\,A)(3.681\,\Omega)}{1.51\,A}$$

$$= 0.2925\,\Omega$$

(c) A 311 Ω resistor is made of three identical resistors ($R = R_1 = R_2 = R_3$) as shown. Find R.

Same Circuit:





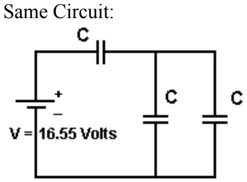
Parallel *Series*

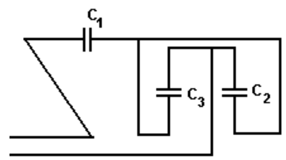
$$\frac{1}{R_{23}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \qquad R_{123} = R + \frac{R}{2} = \tfrac{3}{2}R = 311\Omega$$

$$R = \frac{2(311\Omega)}{3} = 207.3\Omega$$

(d) A 311 pF capacitor is made of three identical capacitors ($C = C_1 = C_2 = C_3$) as shown. Find C.

Same Circuit:





Parallel *Series*

$$C_{23} = C + C = 2C \qquad \frac{1}{C_{123}} = \frac{1}{C} + \frac{1}{2C} = \frac{3}{2C}$$

$$C_{123} = \tfrac{2}{3}C$$

$$C = \tfrac{3}{2}C_{123} = \frac{3(311pF)}{2} = 466.5pF$$

(e) A particular circuit generates 125 W of Joule heating when hooked up to a perfect battery which has a potential difference of 6.00 volts. If the perfect battery is replaced with another perfect battery of half the voltage, find the power dissipated by Joule heating in this new circuit.

$$P = \frac{V^2}{R} \quad ; \quad R = \frac{V^2}{P} = \frac{(6.00volts)^2}{125W} = 0.2880\Omega$$

$$P = \frac{V_2^2}{R} = \frac{(3.00volts)^2}{125W} = 31.25W$$

Just A Couple of Easy Star Problems – Nothing to Worry About (50,000 points)

2.) ☆(a) A capacitor *C* is placed in a circuit where the current leading to the capacitor is given by $I(t) = I_0 \sin(\omega t)$. Find the charge on the capacitor as a function of time, *q(t)*.

$$i = \frac{dq}{dt} \quad ; \quad dq = i\, dt$$
$$q = \int dq = \int i\, dt = \int I_0 \sin(\omega t)\, dt$$
$$q(t) = \frac{-I_0 \cos(\omega t)}{\omega} + C$$

☆(b) An electron enters a velocity selector at its designed speed *v*. It has velocity $\vec{v} = v\,\hat{i}$, E-field $\vec{E} = E\,\hat{j}$ and B-field $\vec{B} = B\,\hat{k}$. Find the work, $W = \int_a^b \vec{F} \cdot d\vec{s}$, done by the electric force on the electron.

$$W = \int_a^b \vec{F} \cdot d\vec{s}$$
$$d\vec{s} = dx\,\hat{i}$$
$$\vec{F}_E = -e\,E\,\hat{j}$$
$$\vec{F}_E \cdot \hat{i} = 0$$
$$So\ldots$$
$$W = 0$$

☆(c) It takes work to store energy in a capacitor as it charges. Integrate $W = \int_0^Q \Delta V\, dq$ to find this energy.

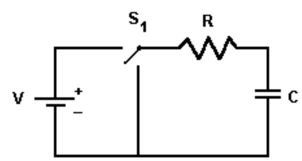
$$C = \frac{Q}{V} = \frac{Q}{\Delta V} \quad ; \quad \Delta V = \frac{Q}{C} = \frac{q}{C}$$
$$W = \int_0^Q \Delta V\, dq = \int_0^Q \frac{q}{C} dq$$
$$= \frac{1}{C}\int_0^Q q\, dq = \frac{1}{C}\frac{q^2}{2}\Big|_0^Q = \frac{Q^2}{2C} - 0 = \frac{Q^2}{2C}$$

(d) At the magnetic North Pole, the B-field lines point straight down. An electron ends up going around and around in a horizontal circle of radius *r = 1.00 cm*. Find the speed *v* of the electron.

Given: $B = 1.00\ Gauss = 1.00 \times 10^{-4}\ T$

$$F_B = F_c$$
$$qvB = ma_c = \frac{mv^2}{r}$$
$$qB = \frac{mv}{r}$$
$$v = \frac{qBr}{m} = \frac{(1.602 \times 10^{-19}\,C)(1.00 \times 10^{-4}\,T)(0.0100m)}{(9.11 \times 10^{-31}\,kg)}$$
$$= 175{,}900\, m/s$$

☆(e) Consider an RC circuit as shown. Flip the switch S_1 to the left and the capacitor charges. Once charged with a charge $\pm Q_0$, if the switch S_1 is turned down, then the capacitor discharges. For the discharging capacitor, write a Kirchhoff's Law voltage loop equation. Remember that the definition of current is *I = dq/dt*. Show that $Q(t) = Q_0\, e^{-t/RC}$.



FOR EITHER CHARGING OR DISCHARGING CAPACITOR, SEE YOUR NOTES…