

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers

Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations

Feel Free to Ask Any Questions

 ☆2a ☆2b ☆2c ☆2e

EXAM 3 [FORM - A] PHYS-207 (KALDON-8)

SPRING 2003

WMU

$$k = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A}$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

Motor Circuit City (50,000 points)

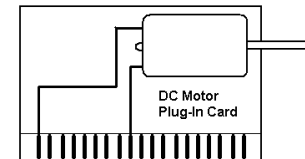
1.) A small DC motor is mounted on a printed circuit card. The copper coil inside the motor consists of 10,000 turns wrapped around a cylindrical core of radius 1.00 cm and length 2.00 cm.

(a) Find the self-inductance, L , of this motor coil. *If you don't get an answer to (a), use $L = 8.43 \text{ H}$ in all subsequent parts.*

$$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}})(10,000)^2 (\pi (0.0100\text{m})^2)}{(0.0200\text{m})}$$

$$= 1.974 \text{ H}$$



(b) The thin motor winding wire has a resistance of 85.0Ω . (Aren't you glad that Dr. Phil didn't make you calculate that as well?) Treat the motor as part of a series RL circuit. When the motor is connected to a battery and turned on, find the time it takes for the current to get up to approximately 95% of the maximum current.

THE SHORT WAY

$$\tau_{RL} = \frac{L}{R} = \frac{1.974 \text{ H}}{85.0 \Omega} = 0.02322 \text{ sec}$$

when $t = 3\tau$, $i(t) \approx 95\% I_{\max}$

$$3\tau = 3(0.02322 \text{ sec}) = 0.06966 \text{ sec}$$

THE LONG WAY

$$\tau_{RL} = \frac{L}{R} = \frac{1.974 \text{ H}}{85.0 \Omega} = 0.02322 \text{ sec}$$

when $t = 3\tau$, $i(t) \approx 95\% I_{\max}$

$$3\tau = 3(0.02322 \text{ sec}) = 0.06966 \text{ sec}$$

$$i(t) = I_0 (1 - e^{-t/\tau})$$

$$0.950 I_0 = I_0 (1 - e^{-t/\tau})$$

$$0.950 = 1 - e^{-t/\tau}$$

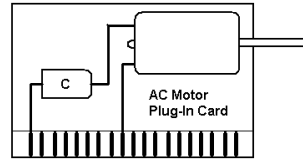
$$e^{-t/\tau} = 1 - 0.950 = 0.050$$

$$\ln(e^{-t/\tau}) = \ln(0.050)$$

$$-\frac{t}{\tau} = -2.996$$

$$t = 2.996 \tau = 0.06957 \text{ sec}$$

A similar AC motor, with the same R and L, is mounted on a second printed circuit card. A capacitor C is mounted in series with the motor, so that the impedance Z will be a minimum when the AC frequency is exactly 60.0 Hz. (c) What is C? (d) What is Z?



(c)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For Z_{\min} , $X_L - X_C = 0$

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C} ; \omega = 2\pi f = 2\pi(60.0\text{Hz}) = 377.0\text{rad/sec}$$

$$C = \frac{1}{\omega^2 L} = \frac{1}{(377.0\text{rad/sec})^2 (1.974\text{H})} = 3.564 \times 10^{-6} \text{F}$$

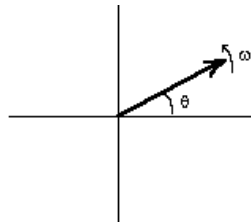
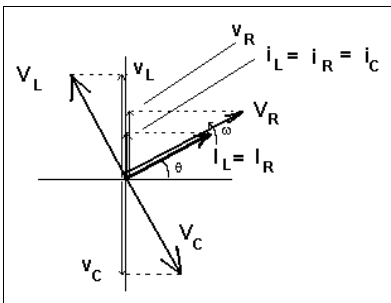
(d)

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For Z_{\min} , $X_L - X_C = 0$

$$Z_{\min} = \sqrt{R^2} = R = 85.0\Omega$$

(e) If the vector in the phasor diagram at the right represents I_{\max} , sketch in V_R , V_L , V_C and show the relevant instantaneous components, with labels.



The All-New Star Search... Hosted by Arsenio Hall (50,000 points)

2.) ☆(a) Two wires, marked 1 and 2 as shown below, each 2.50 m long, are separated by a distance $d = 0.110 \text{ m}$. They carry currents $i_1 = 15.0 \text{ A}$ and $i_2 = 27.0 \text{ A}$. Use Ampère's Law to find the magnitude of \vec{B}_1 at the point where wire 2 is located directly below wire 1. Do the integration properly if you want full credit.



$$\oint \vec{B}_1 \cdot d\vec{s} = \mu_0 I_{\text{enclosed}} ; ds = r d\theta$$

$$\int_0^{2\pi} B_1 r d\theta = B_1 r \int_0^{2\pi} d\theta = \mu_0 i_1$$

$$B_1 r d\theta \Big|_0^{2\pi} = B_1 r (2\pi - 0) = \mu_0 i_1 ; r = d$$

$$B_1 = \frac{\mu_0 i_1}{2\pi d} = \frac{(4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}})(15.0 \text{ A})}{2\pi(0.110\text{m})}$$

$$= 2.727 \times 10^{-5} \text{ T}$$

☆(b) An inductor has, by definition, an inductance $L = \frac{N\Phi_B}{I}$. Use the definition of magnetic flux and do the integral, to show that $L = \frac{\mu_0 N^2 A}{l}$ for an air-core solenoid of radius r and length l .

$$L = \frac{N\Phi_B}{I} ; \Phi_B = \int \vec{B} \cdot d\vec{A} ; dA = r dr d\theta ; B = \frac{\mu_0 N I}{l}$$

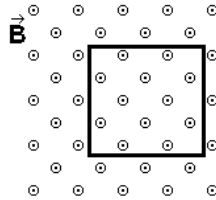
$$\Phi_B = \int B dA = B \int_0^r \int_0^{2\pi} r dr d\theta = B \frac{r^2}{2} \Big|_0^r \theta \Big|_0^{2\pi} = B(\pi r^2)$$

$$L = \frac{NB(\pi r^2)}{I} = \frac{N(A)}{I} \cdot \frac{\mu_0 N I}{l} = \frac{\mu_0 N^2 A}{l}$$

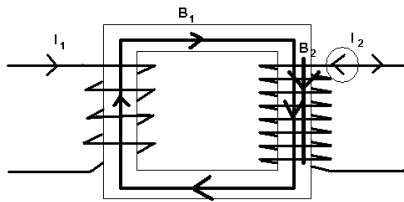
$$L = \frac{\mu_0 N^2 A}{l}$$

★(c) Consider a square in the xy -plane with sides of length a and a magnetic field, $\vec{B} = (Cxy^2)\hat{k}$ where C is a constant. Find the magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$. The lower left corner of the square is at the origin, $x=y=0$.

$$\begin{aligned} \Phi_B &= \int \vec{B} \cdot d\vec{A} = \int B dA = \int_0^a \int_0^a Cxy^2 dx dy \\ &= C \int_0^a x dx \int_0^a y^2 dy \\ &= C \left(\frac{x^2}{2} \Big|_0^a \right) \left(\frac{y^3}{3} \Big|_0^a \right) = C \left(\frac{a^2}{2} \right) \left(\frac{a^3}{3} \right) = C \frac{a^5}{6} \end{aligned}$$



(d) An A.C. transformer is shown below. If at some time t_0 that the primary current I_1 is decreasing and going to the right as shown, then explain whether the induced current of the secondary, I_2 , goes to the LEFT or to the RIGHT. You should sketch in the relevant magnetic field B_1 from the primary coil, and the induced magnetic field B_2 from the secondary coil. Finally, if $V_{1,rms} = 117$ Volts, then find $V_{2,rms}$.



The B -field from I_1 goes up from the center of the coil and then stays in the iron frame, going clockwise. The B -field goes DOWN through coil 2 and is DECREASING, so that coil 2 must create an induced B -field that goes DOWN and therefore, RHR has the current I_2 going to the LEFT as it comes into the top of coil 2.

$$\begin{aligned} N_1 &= 4 \quad ; \quad N_2 = 8 \\ \frac{V_2}{V_1} &= \frac{N_2}{N_1} \\ V_{2,rms} &= \frac{N_2}{N_1} V_{1,rms} = \left(\frac{8}{4} \right) (117 \text{ volts}) = 234 \text{ volts} \end{aligned}$$

★(e) A circular wire of radius r is placed in a magnetic field $\vec{B} = (1.00T / \text{sec}^2) t^2 \hat{k}$. Use the generalized form of Faraday's law, $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$, to find the magnitude of the induced Electric field E in the wire.

$$\begin{aligned} \vec{B} &= (1.00T / \text{sec}^2) t^2 \hat{k} \\ \oint \vec{E} \cdot d\vec{s} &= -\frac{d\Phi_B}{dt} \\ \oint E ds &= -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} \int B dA \\ E \int_0^{2\pi} r d\theta &= -\frac{d}{dt} [B \int dA] = -\frac{d}{dt} [B \int_0^r \int_0^{2\pi} r dr d\theta] \\ E r \int_0^{2\pi} d\theta &= -\frac{d}{dt} [B \int_0^r r dr \int_0^{2\pi} d\theta] \\ E r 2\pi &= -\frac{d}{dt} \left[B \left(\frac{r^2}{2} \right) (2\pi) \right] \\ E (2\pi r) &= -(\pi r^2) \frac{d}{dt} [(1.00T / \text{sec}^2) t^2] \\ E &= -\frac{r}{2} (1.00T / \text{sec}^2) (2t) = -(1.00T / \text{sec}^2) r t \end{aligned}$$

