PHYS-207 (8) (Kaldon-52328) **Name** S O L U T I O N

WMU - Spring 2003

Exam 3A - 100,000 points + 20,000 ☆ points

04/02/2003•Rev.A6

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers

Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations

Feel Free to Ask Any Questions ☐☆2a ☐☆2b ☐☆2c ☐☆2e

EXAM 3 [FORM - A]
PHYS-207 (KALDON-8)

SPRING 2003

WMU

$$k = 8.988\times10^{9}\ N\cdot m^2/C^2$$

$$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$$

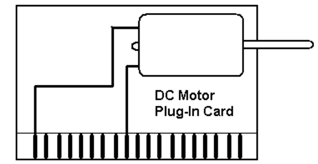
$$\mu_0 = 4\pi\times10^{-7}\ T\cdot m/A$$

$$e = 1.602\times10^{-19}\,C$$

$$m_e = 9.11\times10^{-31}\,kg$$

Motor Circuit City (50,000 points)

1.) A small DC motor is mounted on a printed circuit card. The copper coil inside the motor consists of 10,000 turns wrapped around a cylindrical core of radius 1.00 cm and length 2.00 cm. (a) Find the self-inductance, L, of this motor coil. *If you don't get an answer to (a), use L = 8.43 H in all subsequent parts.*



$$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{\left(4\pi\times10^{-7}\ \tfrac{T\cdot m}{A}\right)(10{,}000)^2\left(\pi(0.0100m)^2\right)}{(0.0200m)}$$

$$= 1.974H$$

(b) The thin motor winding wire has a resistance of 85.0 Ω. (Aren't you glad that Dr. Phil didn't make you calculate that as well?) Treat the motor as part of a series RL circuit. When the motor is connected to a battery and turned on, find the time it takes for the current to get up to approximately 95% of the maximum current.

THE SHORT WAY

$$\tau_{RL} = \frac{L}{R} = \frac{1.974H}{85.0\Omega} = 0.02322\,\text{sec}$$

$$when\ t = 3\tau,\ i(t) \approx 95\%\ I_{\max}$$

$$3\tau = 3(0.02322\,\text{sec}) = 0.06966\,\text{sec}$$

THE LONG WAY

$$\tau_{RL} = \frac{L}{R} = \frac{1.974H}{85.0\Omega} = 0.02322\,\text{sec}$$

$$when\ t = 3\tau,\ i(t) \approx 95\%\ I_{\max}$$

$$3\tau = 3(0.02322\,\text{sec}) = 0.06966\,\text{sec}$$

$$i(t) = I_0\left(1 - e^{-t/\tau}\right)$$

$$0.950 I_0 = I_0\left(1 - e^{-t/\tau}\right)$$

$$0.950 = 1 - e^{-t/\tau}$$

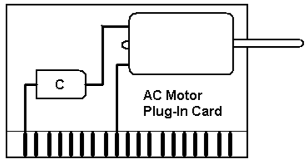
$$e^{-t/\tau} = 1 - 0.950 = 0.050$$

$$\ln\left(e^{-t/\tau}\right) = \ln(0.050)$$

$$-\frac{t}{\tau} = -2.996$$

$$t = 2.996\,\tau = 0.06957\,\text{sec}$$

A similar AC motor, with the same R and L, is mounted on a second printed circuit card. A capacitor C is mounted in series with the motor, so that the impedance Z will be a minimum when the AC frequency is exactly 60.0 Hz. (c) What is C? (d) What is Z?



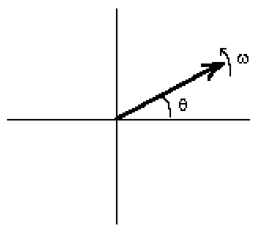
(c)

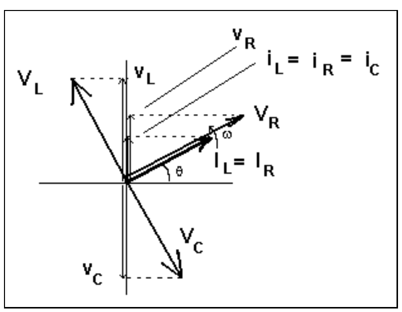
$$Z=\sqrt{R^2+(X_L-X_C)^2}$$
$$For\ Z_{\min},\ X_L-X_C=0$$
$$X_L=X_C$$
$$\omega L=\frac{1}{\omega C}\ ;\ \omega=2\pi f=2\pi(60.0Hz)=377.0 rad/\sec$$
$$C=\frac{1}{\omega^2 L}=\frac{1}{(377.0 rad/\sec)^2(1.974H)}=3.564\times10^{-6}F$$

(d)

$$Z=\sqrt{R^2+(X_L-X_C)^2}$$
$$For\ Z_{\min},\ X_L-X_C=0$$
$$Z_{\min}=\sqrt{R^2}=R=85.0\Omega$$

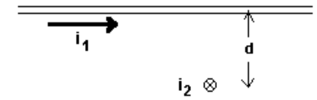
(e) If the vector in the phasor diagram at the right represents I_{max}, sketch in V_R, V_L, V_C and show the relevant instantaneous components, with labels.





The All-New Star Search… *Hosted by Arsenio Hall* (50,000 points)

2.) ☆(a) Two wires, marked 1 and 2 as shown below, each 2.50 m long, are separated by a distance $d = 0.110\ m$. They carry currents $i_1 = 15.0\ A$ and $i_2 = 27.0\ A$. Use Ampère's Law to find the magnitude of $\vec{B}_1$ at the point where wire 2 is located directly below wire 1. *Do the integration properly if you want full credit.*



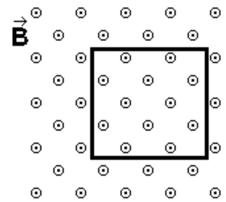
$$\oint \vec{B}_1\cdot d\vec{s}=\mu_0 I_{enclosed}\ \ ;\ \ ds=r\,d\theta$$
$$\int_0^{2\pi}B_1\,r\,d\theta=B_1 r\int_0^{2\pi}d\theta=\mu_0\,i_1$$
$$B_1 r\,d\theta\Big|_0^{2\pi}=B_1 r(2\pi-0)=\mu_0\,i\ \ \ ;\ r=d$$
$$B_1=\frac{\mu_0\,i}{2\pi d}=\frac{(4\pi\times10^{-7}\ \frac{T\cdot m}{A})(15.0A)}{2\pi(0.110m)}$$
$$=2.727\times10^{-5}T$$

☆(b) An inductor has, by definition, an inductance $L=\frac{N\Phi_B}{I}$. Use the definition of magnetic flux and *do* the integral, to show that $L=\frac{\mu_0 N^2 A}{l}$ for an air-core solenoid of radius *r* and length *l*.

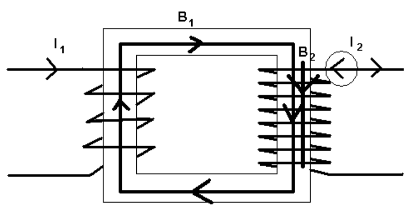
$$L=\frac{N\Phi_B}{I}\ ;\ \Phi_B=\int\vec{B}\cdot d\vec{A}\ ;\ dA=r\,dr\,d\theta\ ;\ B=\frac{\mu_0 N I}{l}$$
$$\Phi_B=\int B\,dA=B\int_0^r\int_0^{2\pi}r\,dr\,d\theta=B\frac{r^2}{2}\Big|_0^r\ \theta\Big|_0^{2\pi}=B(\pi r^2)$$
$$L=\frac{NB(\pi r^2)}{I}=\frac{N(A)}{I}\cdot\frac{\mu_0 N I}{l}=\frac{\mu_0 N^2 A}{l}$$
$$L=\frac{\mu_0 N^2 A}{l}$$

☆(c) Consider a square in the *xy*-plane with sides of length *a* and a magnetic field, $\vec{B} = (C\,xy^2)\,\hat{k}$ where *C* is a constant. Find the magnetic flux $\Phi_B = \int \vec{B}\cdot d\vec{A}$. *The lower left corner of the square is at the origin, x=y=0.*

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B dA = \int_0^a \int_0^a Cxy^2 dxdy$$
$$= C\int_0^a x\,dx \int_0^a y^2\,dy$$
$$= C\left(\frac{x^2}{2}\Big|_0^a\right)\left(\frac{y^3}{3}\Big|_0^a\right) = C\left(\frac{a^2}{2}\right)\left(\frac{a^3}{3}\right) = C\frac{a^5}{6}$$



(d) An A.C. transformer is shown below. If at some time t_0 that the primary current I_1 is <u>decreasing</u> and going to the right as shown, then explain whether the induced current of the secondary, I_2, goes to the LEFT or to the RIGHT. You should sketch in the relevant magnetic field B_1 from the primary coil, and the induced magnetic field B_2 from the secondary coil. Finally, if $V_{1,\,rms} = 117$ *Volts*, then find $V_{2,\,rms}$.



The B-field from I_1 goes up from the center of the coil and then stays in the iron frame, going clockwise. The B-field goes DOWN through coil 2 and is DECREASING, so that coil 2 must create an induced B-field that goes DOWN and therefore, RHR has the current I_2 going to the LEFT as it comes into the top of coil 2.

$$N_1 = 4 \;\;;\;\; N_2 = 8$$
$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$
$$V_{2,rms} = \frac{N_2}{N_1}V_{1,rms} = \left(\frac{8}{4}\right)(117volts) = 234volts$$

☆(e) A circular wire of radius *r* is placed in a magnetic field $\vec{B} = (1.00T/\sec^2)\,t^2\,\hat{k}$. Use the generalized form of Faraday's law, $\oint \vec{E}\cdot d\vec{s} = -\frac{d\,\Phi_B}{d\,t}$, to find the magnitude of the induced Electric field *E* in the wire.

$$\vec{B} = (1.00T/\sec^2)\,t^2\,\hat{k}$$
$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\,\Phi_B}{d\,t}$$
$$\oint E\,ds = -\frac{d}{d\,t}\int \vec{B}\cdot d\vec{A} = -\frac{d}{d\,t}\int B\,dA$$
$$E\int_0^{2\pi} r d\theta = -\frac{d}{d\,t}\left[B\int dA\right] = -\frac{d}{d\,t}\left[B\int_0^r\int_0^{2\pi} r dr d\theta\right]$$
$$E\,r\int_0^{2\pi} d\theta = -\frac{d}{d\,t}\left[B\int_0^r r dr\int_0^{2\pi} d\theta\right]$$
$$E\,r2\pi = -\frac{d}{d\,t}\left[B\left(\frac{r^2}{2}\right)(2\pi)\right]$$
$$E(2\pi r) = -(\pi r^2)\frac{d}{d\,t}\left[(1.00T/\sec^2)\,t^2\right]$$
$$E = -\frac{r}{2}(1.00T/\sec^2)(2\,t) = -(1.00T/\sec^2)\,r\,t$$