PHYS-207 (8) (Kaldon-52328) Name S O L U T I O N

WMU - Spring 2003

Final Exam - 200,000 points + 40,000 ☆ points ☐ You Must Go Through "Check-Out" With Dr. Phil

04/10/2003•Rev.A7

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions ☐☆2a ☐☆2b ☐☆2c ☐☆2e

Check-Out: ☐ Q ☐ X3 Grades ☐ T ☐ ___

FINAL EXAM [FORM - A]
PHYS-207 (KALDON-8)

SPRING 2003

WMU

$$k = 8.988 \times 10^9 \, N \cdot m^2 / C^2$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \, C^2 / N \cdot m^2$$

$$\mu_0 = 4\pi \times 10^{-7} \, T \cdot m / A$$

$$e = 1.602 \times 10^{-19} \, C$$

$$m_e = 9.11 \times 10^{-31} \, kg$$

Consider the Poor Nitrogen Ion… (50,000 points)

1.) A positively charged nitrogen ion ($q = +e$; $m = 14 \times 1.67 \times 10^{-27}$ kg) starts at rest and is driven to a final speed v by an accelerating potential $\Delta V = 6{,}000{,}000$ *volts*. (a) What is the final kinetic energy of the ion?

$$\begin{aligned}\Delta K = K_f - K_i = K_f &= q\Delta V = qV = eV \\ &= 6{,}000{,}000 eV \quad OR \\ &= \left(1.602 \times 10^{-19} C\right)\left(6{,}000{,}000 V\right) \\ &= 9.612 \times 10^{-13} J\end{aligned}$$

(b) If the ion travels 15.0 meters during its acceleration, find the magnitude of the constant electric field E that accelerates the ion.

$$\begin{aligned}&F_E = qE = eE \\ &W = Fd = eEd = 9.612 \times 10^{-13} J \quad OR \\ &qV = qEd \quad OR \\ &V = Ed \\ &E = \frac{V}{d} = \frac{6{,}000{,}000 V}{15.0 m} = 400{,}000 V/m\end{aligned}$$

(c) Suppose that 1.123×10^{15} ions are produced every minute. Express the beam current in Amperes.

$$\begin{aligned}q &= \pm Ne = \left(1.123 \times 10^{15}\right)\left(1.602 \times 10^{-19} C\right) \\ &= 0.0001799 C = 1.799 \times 10^{-4} C \\ I &= \frac{\Delta q}{\Delta t} = \frac{1.799 \times 10^{-4} C}{60.0 \sec} = 2.998 \times 10^{-6} A\end{aligned}$$

(d) This ion is moving in the $+x$ direction when a magnetic field $\vec{B} = 5.00T\,\hat{j}$ is turned on. What is the magnitude and direction of the magnetic force on the ion at the instant the field is turned on?

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(9.612\times10^{-13}J)}{14\times1.67\times10^{-27}kg}} = 9{,}068{,}000m/s$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$F_B = qvB = (1.602\times10^{-19}C)(9{,}068{,}000m/s)(5.00T)$$
$$= 7.263\times10^{-12}N$$
$$direction\ = \hat{i}\times\hat{j} = \hat{k}\ by\ RHR$$
$$\vec{F}_B = 7.263\times10^{-12}N\ \hat{k}$$

(e) What is the period of the resulting circular motion?

$$F_B = F_c$$
$$qvB = \frac{mv^2}{r}$$
$$qB = \frac{mv}{r}$$
$$r = \frac{mv}{qB}$$

$$v = \frac{d}{t} = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}$$
$$= \frac{2\pi(14\times1.67\times10^{-27}kg)}{(1.602\times10^{-19}C)(5.00T)} = 1.834\times10^{-7}\sec$$

Final Star Points (50,000 points)

2.) ☆(a) An electric field is given by $\vec{E} = (5250\,{}^{V}\!/_{m^2})\,x\,\hat{i}$. Use the integral definition of potential difference to find ΔV from the origin (x = 0, y = 0) to (x = 0.250 m, y = 0).

$$V = -\int \vec{E}\cdot d\vec{s}$$
$$= -\int_0^{0.250m} E\,dx = -\int_0^{0.250m}\left((5250\,{}^{V}\!/_{m^2})\,x\right)dx$$
$$= -(5250\,{}^{V}\!/_{m^2})\int_0^{0.250m} x\,dx = -(5250\,{}^{V}\!/_{m^2})\left.\frac{x^2}{2}\right|_0^{0.250m}$$
$$= -(5250\,{}^{V}\!/_{m^2})\frac{(0.250m)^2}{2} = 656.3V$$

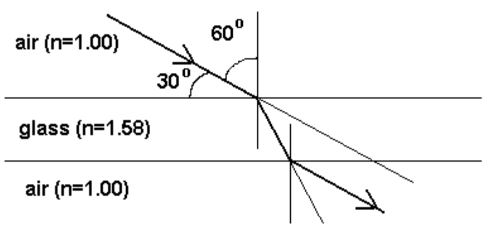
☆(b) Show if B = [$A_1 t + B_2\cos(kx - \omega t)$] is a solution to the wave equation for B, $\frac{\partial^2 B}{\partial t^2} = c^2\frac{\partial^2 B}{\partial x^2}$.

$$B = \left[A_1 t + B_2\cos(kx-\omega t)\right]$$
$$\frac{\partial^2 B}{\partial t^2} = c^2\frac{\partial^2 B}{\partial x^2}$$
$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial t}\left[A_1 t + B_2\cos(kx-\omega t)\right] = A_1 - (-\omega)B_2\sin(kx-\omega t)$$
$$= A_1 + \omega B_2\sin(kx-\omega t)$$
$$\frac{\partial^2 B}{\partial t^2} = \frac{\partial}{\partial t}\left[\frac{\partial B}{\partial t}\right] = \frac{\partial}{\partial t}\left[A_1 + \omega B_2\sin(kx-\omega t)\right]$$
$$= 0 - \omega^2 B_2\cos(kx-\omega t)$$
$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x}\left[A_1 t + B_2\cos(kx-\omega t)\right] = 0 - k B_2\sin(kx-\omega t)$$
$$\frac{\partial^2 B}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{\partial B}{\partial x}\right] = \frac{\partial}{\partial x}\left[-k B_2\sin(kx-\omega t)\right]$$
$$= 0 - k^2 B_2\cos(kx-\omega t)$$
$$\frac{\partial^2 B}{\partial t^2} = c^2\frac{\partial^2 B}{\partial x^2}$$
$$-\omega^2 B_2\cos(kx-\omega t) = -c^2k^2 B_2\cos(kx-\omega t)$$
$$\omega^2 = c^2k^2$$
$$c^2 = \frac{\omega^2}{k^2} = \frac{(2\pi f)^2}{\left(2\pi/\lambda\right)^2} = (f\lambda)^2 = c^2 \quad ;\ YES$$

☆(c) It takes work to store energy in a capacitor as it charges. Integrate $W = \int_0^Q \Delta V \, dq$ to find this energy.

$$C = \frac{Q}{V} = \frac{Q}{\Delta V} \quad ; \quad \Delta V = \frac{Q}{C} = \frac{q}{C}$$
$$W = \int_0^Q \Delta V \, dq = \int_0^Q \frac{q}{C} dq$$
$$= \frac{1}{C}\int_0^Q q \, dq = \frac{1}{C}\frac{q^2}{2}\Bigg|_0^Q = \frac{Q^2}{2C} - 0 = \frac{Q^2}{2C}$$

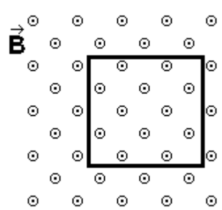
(d) A perfectly flat piece of glass (known as parallel-plano in the optics business) has air on both sides of it as shown. Calculate and sketch on the diagram what happens to the twice refracted light ray when it gets back into the air. *Ignore all reflections.*



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$
$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$
$$= \sin^{-1}\left(\frac{1.00 \sin 60^\circ}{1.58}\right) = 33.2^\circ$$

☆(e) Consider a square in the *xy*-plane with sides of length *a* and a magnetic field, $\vec{B} = (Cxy^2)\,\hat{k}$ where *C* is a constant. Find the magnetic flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$. *The lower left corner of the square is at the origin, x=y=0.*

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\,dA = \int_0^a \int_0^a Cxy^2 \, dx\,dy$$
$$= C\int_0^a x\,dx \int_0^a y^2\,dy$$
$$= C\left(\frac{x^2}{2}\Bigg|_0^a\right)\left(\frac{y^3}{3}\Bigg|_0^a\right) = C\left(\frac{a^2}{2}\right)\left(\frac{a^3}{3}\right) = C\frac{a^5}{6}$$



Problems of an Electrical and Magnetic Nature (50,000 points)

3.) A deuteron (the nucleus of heavy hydrogen, $m = 3.344 \times 10^{-27}$ *kg* ; $q = +e$) is projected towards a uranium nucleus ($q = +92\,e$). When the deuteron is far from the nucleus its speed is 8.00×10^5 m/s. If the uranium nucleus can be regarded as fixed in place, how close does the deuteron get to the nucleus?

Using Work and Energy:

$$\Delta K = q\Delta V$$
$$K_f - K_i = -K_i = q(V_f - V_i) = qV_f$$
$$-\tfrac{1}{2}mv^2 = q\left(-\frac{kQ}{r}\right) = -\frac{kqQ}{r}$$
$$r = \frac{2kqQ}{mv^2} = \frac{2ke(92e)}{mv^2} = \frac{184ke^2}{mv^2}$$
$$= \frac{184\left(8.988 \times 10^9 \, N \cdot m^2/C^2\right)\left(1.602 \times 10^{-19}\,C\right)^2}{\left(3.344 \times 10^{-27}\,kg\right)\left(8.00 \times 10^5\,m/s\right)^2}$$
$$= 0.00001588m = 1.588 \times 10^{-5} m$$

(b) A spherical Gaussian surface of radius 2R is placed around a solid conducting metal sphere of total charge Q and radius R. Find the total electric flux that passes through the Gaussian surface.

$$\Phi_E = \oint_{surface} \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

(c) A spherical Gaussian surface of radius 2R is placed around an insulating sphere of total charge Q spread and volume charge density ρ and radius R. Find the total electric flux that passes through the Gaussian surface.

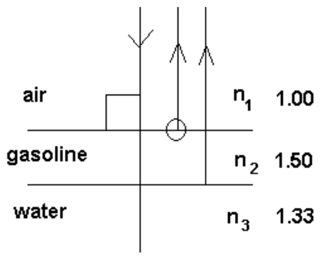
We're outside the sphere, so SAME answer as (b).

(d) A cylindrical Gaussian surface of radius R and length L is placed around a long straight wire of current I. Find the total magnetic flux that passes through the Gaussian surface.

$$\Phi_B = \oint_{surface} \vec{B} \cdot d\vec{A} = 0$$

(e) At a gas station you happen to look straight down at a puddle of water ($n = 1.33$) that has a thin layer of gasoline ($n = 1.50$) on top. Although white sunlight is shining straight down on the puddle, the reflection contains absolutely no red light ($\lambda = 640.\ nm = 640. \times 10^{-9}\ m$). What is the minimum thickness of the gasoline layer?

The first reflection is low-to-high while the second is high-to-low, so only one is phase-shifted half a wavelength, and therefore the round trip distance must be an integer wavelength.



$$\lambda_m = \frac{\lambda}{n_m} = \frac{640nm}{1.50} = 426.7nm$$
$$d = 2t = \lambda_m$$
$$t = \frac{\lambda_m}{2} = \frac{426.7nm}{2} = 213.4nm$$

Physics Takes A Circuitous Path… And It All Comes Out Here (50,000 points)

4.) You are given a resistor R = 100.Ω, a capacitor C = 1000. μF and an inductor L = 0.100 H. The capacitor has parallel plates of sides *a* and a gap $d = 1.00\ mm$. Find *a*.

$$C = \varepsilon_0 \frac{A}{d} = \varepsilon_0 \frac{a^2}{d}$$
$$a^2 = \frac{Cd}{\varepsilon_0}$$
$$a = \sqrt{\frac{Cd}{\varepsilon_0}} = \sqrt{\frac{(1000.\times 10^{-6} F)(0.00100m)}{(8.85\times 10^{-12} F/m)}}$$
$$= 336.1m$$

(b) The inductor is a solenoid of 1000. turns and a diameter $D = 6.00\ mm$. Find the length *l* of the inductor.

$$D = 0.00600m\ ;\ r = 0.00300m$$
$$L = \frac{\mu_0 N^2 A}{l}$$
$$l = \frac{\mu_0 N^2 A}{L}$$
$$= \frac{\left(4\pi\times 10^{-7}\ \tfrac{T\cdot m}{A}\right)(1000)^2\left(\pi (0.0030m)^2\right)}{(0.100H)}$$
$$= 0.0003553m = 3.553\times 10^{-4} m$$

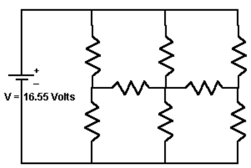
(c) All three of these devices are plugged into a series RLC circuit and run on 50.0 Hz A.C. Find the impedance *Z* of this circuit.

$$\omega = 2\pi f = 2\pi(50.0Hz) = 314.2 rad/\sec$$
$$X_L = \omega L = (314.2 rad/\sec)(0.100H)$$
$$= 31.42\Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(314.2 rad/\sec)(1000.\times 10^{-6} F)}$$
$$= 3.183\Omega$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(100.\Omega)^2 + (31.42\Omega - 3.183\Omega)^2}$$
$$= 103.9\Omega$$

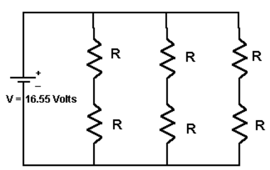
(d) A brand fresh battery has a potential difference $V = 1.50\ volts$ with a 100. Ω resistor attached in a simple circuit. Some time later, the voltage has dropped to 1.37 volts. Find the internal resistance r of the weakened battery.

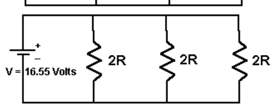
Real Battery = Perfect Battery w/ Internal Resistance

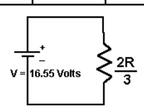
$$V = IR \quad or \quad \mathcal{E} = I(r+R)$$
$$I = \frac{V}{R} = \frac{1.38 volts}{100.\Omega} = 0.01380 A$$
$$\mathcal{E} = I\,r + I\,R$$
$$I\,r = \mathcal{E} - I\,R = \mathcal{E} - V$$
$$r = \frac{\mathcal{E} - V}{I}$$
$$= \frac{1.50 volts - 1.38 volts}{0.01380 A}$$
$$= 8.696\Omega$$



(e) This little festive arrangement has eight identical resistors, each $R = 100.\ \Omega$, as shown. This is another circuit that technically requires Kirchhoff's Laws to solve, but because the resistors are identical, you can easily find the equivalent resistance, R_{eq} , of this circuit. What is R_{eq} ? *Sketch the intermediate circuits and circle the resistors you combining, indicating if they are in series or parallel, if you want full credit for this problem.*







$$R_{12} = R_1 + R_2 = 2R$$
$$\frac{1}{R_{123}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{2R}$$
$$R_{eq} = \frac{2R}{3} = \frac{2(100.\Omega)}{3} = 66.67\Omega$$