Magnesium Hits On The Ol' Mass Spectrometer (25,000 points)

1.) A desktop mass spectrometer is going to be used to separate two magnesium isotopes. (a) Mg$^+$ ions, moving at $v = 125 \text{ m/s}$, safely pass through two velocity selectors. The first has $\vec{B} = -1500 \text{ T} \hat{k}$, the second has $\vec{E} = +1500 \text{ N/C} \hat{k}$. Find the $\vec{E}$ and $\vec{B}$, respectively, of the two velocity selectors.

(b) We design the mass spectrometer so that $^{22}\text{Mg}^+$ ions (mass = $22 \times 1.7 \times 10^{-27} \text{ kg}$) will travel in a semi-circle of diameter $D = 0.305 \text{ m}$ (one foot). Find the magnetic field vector, $\vec{B}$, that will do this.

(c) Use the magnetic field vector, $\vec{B}$, from (b) to determine the diameter of the path of any $^{24}\text{Mg}^+$ ions that happen to enter the mass spectrometer.

(d) What is the separation distance, $d$, between the $^{22}\text{Mg}^+$ and $^{24}\text{Mg}^+$ ions after they have traveled around their semi-circles? Is this separation large enough to measure on film, filter paper or electronic detector?

(e) If a trillion Mg$^+$ ions go through the mass spectrometer every second, what current, $i$, does this represent?

2.) The circuit on the left is the same as the circuit that showed up in Quiz # 9†, except that there is a new resistor at R6. The same circuit is redrawn at the right for clarity. You'd think that you could solve this using network reduction, but trust Dr. Phil, you can't. Given that all the resistors are 125 $\Omega$ however, you can find an equivalent resistance if you realize that the current through R4, $I_4 = 0$. (a) Why is $I_4 = 0$?

(b) Find $R_{\text{TOTAL}}$ and (c) $I_{\text{TOTAL}}$ for the circuit.

(d) Find the currents through all the resistors ($I_1$, $I_2$, $I_3$, $I_5$ and $I_6$) given that $I_4 = 0$.

(e) Treating all the currents through the resistors ($I_1$, $I_2$, $I_3$, $I_5$ and $I_6$) as unknowns, write down but do not solve all the equations you need to solve this circuit as a multi-loop circuit (Kirchhoff’s Laws). Note which ones would show that $I_4 = 0$.

† From PHYS-207 Summer 1995.
Odds & Ends: Mostly Odd, Mostly Math (25,000 points)

3.) \(\textbf{a) }\) An RC circuit has a time constant of \( RC = 1.00 \text{ s} \). If the initial charge was \( q_0 = 1.00 \times 10^{-5} \text{ C} \), find the time \( t \) where the remaining charge is equal to the last electron: \( q(t) = e \). (After this \( \pm e \) charge leaves the two plates, \( q(t) = 0 \), so it does not take an infinite amount of time to discharge a capacitor.)

A spherical conductor of radius \( R = 0.100 \text{ m} \) and charge \( Q = 1.38 \times 10^{-3} \text{ C} \) has the electric field of a point charge outside the sphere, and \( E = 0 \) inside. Find the potential \( V(r) \) for:

\( \textbf{b) } r > R \) and \( \textbf{c) } r < R \), using:

\[ \Delta V = V_f - V_i = \int V \cdot dl \] . Choose \( V = 0 \) at \( r = \infty \).

Finding the resistance \( R \) of a truncated cone of length \( L \) and radii \( a \) and \( b \) (where \( a > b \)), turns out to be a horrible problem. However, assuming that the current density, \( J \), is constant over any circular cross-sectional slice of the object, you can easily write down expressions for (d) the current density \( J \) and

\( \textbf{e) } \) the electric field \( E \), using the current \( i \) and resistivity \( p \). Hint: the radius as a function of \( x \) is:

\[ r(x) = a - \frac{a - b}{L} x , \text{ if you are interested in being really complete...} \]

Batman Returns Forever (25,000 points)

4.) In the movie Batman Returns from a couple of years ago, Michelle Pfeiffer’s character gets into trouble because she saw the plans for a building her evil boss was constructing to “steal all the power from Gotham City” and determined that these plans were for “a giant capacitor”. Consider an air filled parallel plate capacitor that is building sized: length = 100 m, width = 50.0 m, plate separation = 30.0 m. Find (a) the capacitance, \( C \), and charge \( Q \) on the plates if the voltage difference is \( V = 125,000 \text{ volts} \).

(b) Rather than making just one big capacitor to fill a building, suppose we stack a bunch of thin capacitors in the building. These are constructed out of metal foil, with 1.00 mm (.00100 m) thick layer of glass (\( \kappa = 4.7 \); dielectric strength = \( 14 \times 10^6 \text{ V/m} \)) acting as a dielectric between the plates, and a 1.00 mm (0.00100 m) thick layer of insulation between the layers. Find the capacitance of just one of these thin parallel plate capacitors. Use the length and width of the building from above.

(c) Will this capacitor work with \( V = 125,000 \text{ volts} \)? If not, determine what \( V_{\text{MAX}} \) should be.

(d) The building is tall enough to hold 15,000 of these capacitors. Find \( C_{\text{TOTAL}} \) for 15,000 capacitors in series and for 15,000 capacitors in parallel.

(e) Find the energy stored in 15,000 capacitors in series and 15,000 capacitors in parallel, using the \( V_{\text{TOTAL}} = V_{\text{MAX}} \). Which arrangement stores more energy? (By the way, this turns out to be a stupid thing for a criminal to do, because you can only charge the capacitor once – but then who said that comic book villains were smart?)