

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers

Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations

Feel Free to Ask Any Questions ☆2c ☆2d ☆2e ☆4e

The Light Bulb and The Room Problem (25,000 points)

1.) If you look closely at the filament in most light bulbs, you'll see that the wire is usually coiled. If the filament is coiled, then the coil must have an inductance. (a) Find the inductance, L_1 , of the light bulb, given that the filament is 1.00 cm (0.0100 m) long, has a diameter of 4.00 mm (0.00400 m) and has 15 turns.

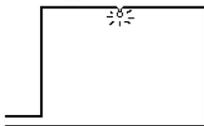


(b) This is a 25 W light bulb, that is it dissipates 25 W of power from its resistance. For $V = 125$ Volts, find the current, i , and resistance, R , of the light bulb.

(c) Find the energy, U_L , that the inductor L_1 can hold in its magnetic field, with the current i .

(d) Although the inductor is made up from the very wire that forms the resistor, we can consider R and L_1 to be in series. Find the time constant of the light bulb. Will it make any difference in how long it takes for the bulb to light? Do you have to state any other assumptions in this part?

(e) Suppose this light bulb in a room is wired so that the current carrying wire makes a big rectangular loop, 3.00 m x 5.00 m. Find the inductance L_2 of this loop. L_1 and L_2 will be in series. Which inductance has the bigger impact on the circuit?



A Tale of Two Wires (25,000 points)

2.) Two wires, marked 1 and 2 as shown below, are each 2.50 m long and are separated by a distance $d = 0.130$ m. They carry currents $i_1 = 4.00$ A and $i_2 = 2.50$ A. Find the forces (a) $\vec{F}_{1 \text{ on } 2}$ and (b) $\vec{F}_{2 \text{ on } 1}$. Indicate whether the forces are attractive or repulsive. *Don't use Newton's 3rd Law!*



☆(c) Use Ampere's Law to find $|\vec{B}_2|$ for $r = d$. Sketch \vec{B}_2 on the diagram below. *Do the integration properly if you want full credit.*



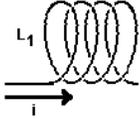
☆(d) Using a cylindrical Gaussian surface and the magnetic field from wire 2, \vec{B}_2 , show that Gauss' Law for Magnetism is satisfied, i.e. that there are no magnetic monopoles present in the wire.

☆(e) If A.C. was running in wire 2, its changing B-field should have an effect on wire 1. Assuming that $\vec{B}_2(r,t)$ is uniform over the small width (0.00500 m) of the wire 1, find the induced electric field, E , in the largest available circular path inside wire 1, due to the changing magnetic field of the 60 Hz A.C. current in wire 2.



Mutual Self-Admiration Society Meeting (25,000 points)

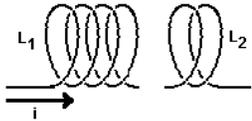
3.) A current $i = 1.00$ A flows through a 400 loop inductor, $L_1 = 44.4$ mH. (a) Find the flux Φ_B of the magnetic field inside.



(b) Do you have enough information to find the magnitude of the B-field given Φ_B from (a)? To find the B-field using the solenoid equation? In each case, either state what else you'd need to know or find B.

(c) If the current was suddenly introduced into the inductor L_1 in 0.0100 s, find the self-induced emf.

(d) Nearby there is a second inductor, $L_2 = \frac{1}{2} L_1$. If L_2 has only 200 loops, find the mutual inductance coefficient, M , between the two coils and the emf induced in L_2 by the current i_1 from (c).



(e) If these two coils were part of a transformer, and the primary (L_1) voltage was 110 V 60 cycle A.C., what would be the voltage on the secondary side?

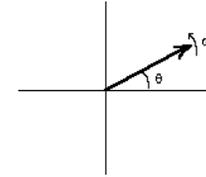
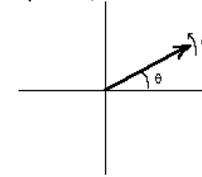
"Phasors Set on Stun, Captain!" (25,000 points)

4.) An RLC circuit contains the following elements: $R = 1000 \Omega$, $C = 1000$ pF and $L = 1000$ mH. It is driven by an AC generator with $\omega = 377$ rad/s. (a) Find the capacitive and inductive reactances, X_C and X_L .

(b) Find the impedance, Z , for the circuit. For what value of ω is the impedance a minimum?

(c) If the vector in the phasor diagram on the left represents I_L , sketch in V_L and show the relevant instantaneous components, i_L and v_L . *Don't worry about the scales of the graphs.*

(d) If the vector in the phasor diagram on the right represents I_R , sketch in V_R and show the relevant instantaneous components, i_R and v_R .



★(e) Show whether or not $q = Q_1 \cos(\omega t) + Q_2 \sin(\omega t)$ is a proper solution to the LC differential

equation: $L \frac{d^2 q}{dt^2} + \frac{1}{C} q = 0$.