

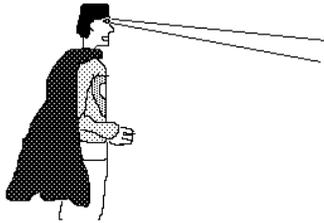
State Any Assumptions You Need To Make -- Show All Work -- Circle Any Final Answers
Use Your Time Wisely - Work on What You Can - Be Sure to Write Down Equations
BOLDFACE Variables Are Vectors - Feel Free to Ask Any Questions

☆2c ☆4a ☆4b ☆4d

The Man of Steel: Defending Truth, Justice and The American Way (40,000 points)

1.) Superman (star of comic books, TV and movies) is supposed to have many special powers, including X-ray vision. X-rays, of course, are both a form of light (electromagnetic radiation) and a dangerous type of radiation. A typical X-ray has a wavelength about the same size as a typical atom: $\lambda = 1.00 \text{ \AA} = 1.00 \times 10^{-10} \text{ m}$. (a) What is the frequency, f , of such an X-ray?

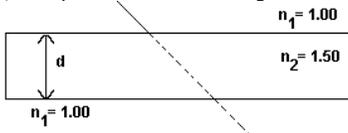
(b) Normal vision is accomplished by focusing visible light reflected from objects onto the retina. If Superman's X-ray vision were to work the same way as normal vision, what would you expect Superman to be able to see?



(c) Superman, of course, has perfectly normal regular vision. However, when he is disguised as Clark Kent, he wears eyeglasses that he doesn't need. If the glasses have a strength of +1.00 diopters (a diopter is the inverse of the focal length in meters; this corresponds to a focal length, $f = +100 \text{ cm}$), what focal length lens do you need to add to cancel the lens in the glasses? What is the focal length of a flat piece of glass?

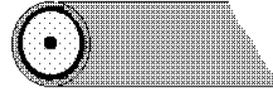


(d) If Superman looks at the flat glass of the phone booth, what happens to a ray that comes in at $\theta = 45^\circ$?



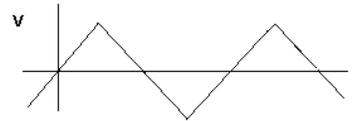
Wires And Waves (40,000 points)

2.) Coaxial wire (coax) consists of a conducting core, a layer of insulation, a layer of conducting wire fabric and an outer layer of insulation. If the core is made of copper ($r = a = 1.00 \text{ mm} = 0.00100 \text{ m}$; $\rho = 1.69 \times 10^{-8} \Omega \cdot \text{m}$) and the insulation is polystyrene ($r = b = 5.00 \text{ mm} = 0.00500 \text{ m}$; $\kappa = 2.6$), find (a) the resistance per meter (R/L) and (b) capacitance per meter (C/L) of this coax. The equation for a cylindrical capacitor is: $C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$.



☆(c) Using the other capacitor equation, $Q = CV$, find the above equation for a cylindrical capacitor by integrating $V = \int E ds$, where E is the electric field of a uniformly charged cylinder (same as a line of charge).

(d) We know that for a sine wave $V_{RMS} = .7071 V_{MAX}$. How would you find V_{RMS} for a triangular sawtooth wave?



Run Maxwell, Run! (40,000 points)

3.) For a traveling E&M wave, Faraday's Law of Induction can be rewritten as a partial differential equation, and Maxwell's Law of Induction is similarly transformed:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \rightarrow \frac{\partial \mathcal{E}}{\partial x} = -\frac{\partial \mathcal{B}}{\partial t} \quad \oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \rightarrow -\frac{\partial \mathcal{B}}{\partial x} = \epsilon_0 \mu_0 \frac{\partial \mathcal{E}}{\partial t}$$

If you take the right hand equations, and differentiate them with respect to t or x, you will end up with terms that you can combine to form the wave equations (a type of 2nd order differential equation),

for E and B: ☆(a) $\frac{\partial^2 \mathcal{E}}{\partial t^2} = c^2 \frac{\partial^2 \mathcal{E}}{\partial x^2}$ and ☆(b) $\frac{\partial^2 \mathcal{B}}{\partial t^2} = c^2 \frac{\partial^2 \mathcal{B}}{\partial x^2}$

(c) For the traveling E&M wave, $E = E_m \cos(kx - \omega t)$ and $B = B_m \cos(kx - \omega t)$, we know that $\omega = 2\pi f$, but what is "k", the wave number equal to? *Hint: if you don't know, consider what dimensions k must have, and what function it serves.*

☆(d) Differentiate $E = E_m \cos(kx - \omega t)$ in the wave equation for E (see above) and see if it is a solution.

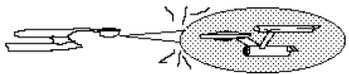
Star Trek Science (40,000 points)

4.) As pointed out in class, not everything in science fiction is scientifically accurate. Take *Star Trek*, for example. The *USS Enterprise* (right) can apply a *tractor beam* on the Klingon warship (left), which applies an attractive force. We can actually make something like a tractor beam, if the *Enterprise* transfers some electrons to the Klingon. (a) If both ships started out as charge neutral, each has a mass of 20,000 metric tons (20×10^6 kg) and are one kilometer (1000 m) apart, find the acceleration between them after -5.00 C of electrons are transferred.

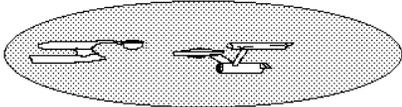


(b) The electrons are accelerated through a potential difference of 2.20×10^6 V. Find the speed of the electrons, using $K = \frac{1}{2}mv^2$. Determine if the final speed of the electrons is relativistic.

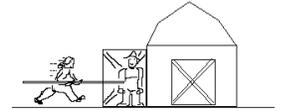
(c) The Klingon warship doesn't like being hit with a tractor beam, so it fires some sort of laser beam at the *Enterprise*. However, the *Enterprise* is protected by some sort of electromagnetic shield. If the laser light undergoes destructive interference everywhere on the surface of the bubble, does this stop the laser beam? Or do the light waves continue on and hit the *Enterprise* anyway? You might want to consider what happens in the double slit interference experiment if you remove the target. **BRIEF ANSWER!**



(d) If the *Enterprise* were to extend its shields to triple the previous radius, how much more power must go to the shields to keep the intensity the same? *Treat the shields as a spherical shell of radius R and $3R$.*

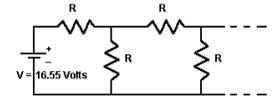
**Miscellaneous (40,000 points)**

5.) Here's a classic special relativity problem: A pole-vaulter carrying a 12 foot long pole. A farmer bets the pole-vaulter \$50 that he can fit the 12 foot long pole into a 10 foot long barn. Sure that he is about to make a quick \$50, the pole-vaulter accepts the bet. The farmer now asks him to run toward the barn at $v = 0.900$ c. (a) Determine if the farmer wins the bet from his point of view.

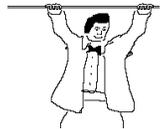


(b) Determine if the pole-vaulter wins the bet from his point of view. Who collects the \$50?

(c) For an infinite system of 100Ω resistors, as shown below, find the equivalent resistance. *Hint: You may need to try some shorter, finite systems first.*



(d) James Bond has a resistance of $.007 \times 10^8 \Omega$ (that's $7.00 \times 10^5 \Omega$). He hangs onto a wire carrying 1500 A of current and has a potential difference of 50,000 V from the ground. His hands are 85 cm apart and the cable has a resistance per length of $1.00 \Omega/m$. Find the current that goes through Mr. Bond.

**Wavy-Gravy (25,000 points)**

5.) Heinrich Hertz's original apparatus for generating and detecting electromagnetic waves has very small values of L and C , where $f \approx 100$ MHz.[†] (a) Suppose Hertz had waves of $f = 111$ MHz. What is the wavelength, λ , of this E&M wave?

(b) Hertz's apparatus created E&M waves when a spark jumps the gap between two spherical conductors along the x -axis. Therefore $E_{max} = 3.00 \times 10^6$ V/m. What is B_{max} to this E_{max} ?

(c) What is the value of the Poynting vector, \vec{S} ? The magnetic field is along the y -axis.

Write down the equations for the traveling waves (d) $E(z,t)$ and (e) $B(z,t)$, including all the relevant variables.

(f) Since this is only a sample problem – what kind of wave are we dealing with anyway?

(g) Our Hertzian wave hits a wall and is completely absorbed. What is the radiation pressure, P , that this wave exerts on the wall?

(h) The Hertzian waves strikes the wall and are completely absorbed for 10.0 seconds. Either find the momentum of the waves, p , or show that you do not have enough information to solve the problem.

[†] Physics: For Scientists and Engineers / Serway & Beichner (5th Edition, ©2000), § 34.1, p. 1077.