

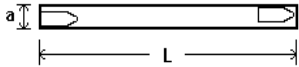
State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions

For all problems, use $c = 2.998 \times 10^8 \text{ m/s}$; $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$; $\hbar = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$.

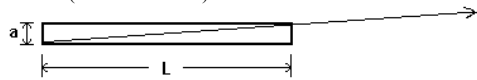
The Galileo-deBroglie-Heisenberg .30-'96 (25,000 points)

1.) Consider a rifle barrel that is 1.000 m long, and uses a 50.0 g bullet (0.0500 kg). The bullet is traveling at 400.0 m/s when it leaves the barrel. (a) Since bullet speed is a function of barrel length, estimate the uncertainty of the speed of the bullet, as proportional to the uncertainty of the barrel length. *Use the last reported decimal place of the barrel length as your measure of barrel length uncertainty.*

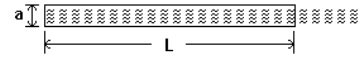
(b) If the bullet is 0.762 cm (0.00762 m) in diameter and the barrel is 0.767 cm in diameter to accommodate the bullet, find the uncertainty of the momentum in the y -direction, Δp_y . *Hint: How much time does it take for the bullet to travel down the barrel?*



In theory, a laser beam is perfectly straight train of photons all with exactly the same frequency ν and the same energy E . Consider a red laser with a wavelength of 640 nm (640×10^{-9} m). (c) Real lasers have a finite length and width, and are subject to beam spread, unless lenses modify the beam. We can establish a classical beam spread just from the angle that a photon might make if it bounces off the lower left corner and emerges at the upper right of the laser tube. What is the spread angle, θ_1 , for a tube that is 0.305 m long and has a diameter of 0.1 mm ($a = 0.000100$ m)?



(d) Now let's look at the same laser tube from a Heisenberg point of view. The photons are coming with a wavelength λ , an energy E , and also a momentum p . Take the tube width, a , as the uncertainty of the position of the photon in the y -direction and use it to find the uncertainty of the momentum of the photon in the y -direction. Find the spread angle, θ_2 , based on this new information. Compare this with the answer in (c).



(e) Use the time the photon spends in making a one-way trip from left to right in the laser tube to find the uncertainty in the energy of any of these photons. Use ΔE to estimate the Δp of the photons. Is this large or small compared to the Δp one might find using the length of the laser tube directly?

Work, Work, Work (25,000 points)

2.) The work function of sodium (Na) is given in Table 2.1 (Serway M&M) as 2.28 eV. (a) Find the frequency of light that would release an electron from a sodium surface with essentially no kinetic energy.

(b) Find the frequency of light that would release an electron from a sodium surface with a speed of 1.00% that of light.

(c) We haven't yet completely talked about how atoms are put together, but the Na atom has 11 electrons. That means the outside electron would be in the $n = 3$ orbital somewhere. Consider a hydrogenic sodium ion, Na^{10+} , where we can use the Bohr atom. Find the energy needed to remove an $n = 3$ electron from Na^{10+} .

(d) Find the frequency of the photon needed in part (c). Does this frequency compare more to the answer in (a) or the answer in (b)? Or do you get a completely different answer?

(e) Find the deBroglie wavelength of the $n = 3$ electron in Na and compare it to the circumference of the orbit.

Bohring Atoms (25,000 points)¹

3.) For real atoms, each set of spectra lines is like a fingerprint identifying one atom as different from another. But this isn't the case with the Bohr model and the hydrogenic ions, because the only variables are Z and n . Consider hydrogenic helium, He^+ . (a) Can you find three cases of Z' and n' in He^+ that have identical radii, r_n , to some r_n of an n -th orbital in hydrogen?

(b) Can you find three cases of Z' and n' in He^+ that have identical energy, E_n , to some E_n of an n -th orbital in hydrogen?

(c) Can you find the frequency and wavelength, f and λ , of an emission photon that is the same in the hydrogen atom, H, and the hydrogenic helium, He^+ . How would you tell the difference between the H spectrum and the He^+ spectrum?

(d) The kinetic energy of an electron in a Bohr atom is $K = -E_n$. (Why?) Use this information to find an equation that starts off $v = \dots$. If $v > 0.417 c$, then $\gamma > 1.10$, and we should really change to relativistic systems. For what element in the periodic table, does the ground state electron have $v \geq 0.417 c$?

¹ As I recall, the students had way too much "fun" with this problem – as in maybe there *aren't* solutions to all parts of (a) - (c).

(e) The ground state electron ($n = 1$) of hydrogenic uranium, U^{91+} , is surely relativistic. Assuming that the energy remains the same, find the speed v of this electron, using both the classical K.E. and the relativistic K.E.

Black Body Radiation and Other Wonders of Science (25,000 points)

$$u(f, T) = \frac{8\pi hf^3}{c^3} \left(\frac{1}{e^{hf/k_B T} - 1} \right)$$

4.) Our take-home quiz² on black body radiation has certainly provided some different answers. You may recall that the power/volume equation was $u(f, T)$, a function of both frequency and temperature. A plot of u , for the temperature $T = 5400K$, is shown below. (a)

Dr. Phil's first attempt to solve the function $\frac{\partial u(f, T)}{\partial f} = 0$ by hand ended up generating the equation: $\frac{hf}{k_B T} = 2$.

Find the wavelength, λ , of the peak by this equation (which does include an expansion of the exponential to just two terms) at $T = 5400K$.

(b) Example 2.2 (Serway M&M) gives us Wien's displacement law $\lambda_{max} T = 2.898 \times 10^{-3} m \cdot K$. Find the wavelength, λ , of the peak by this equation at $T = 5400K$.

² The take-home quiz that semester was to try to find the peak f of black body radiation at $T = 5400 K$, by finding where $\partial u/\partial f = 0$.

(c) Another attempt at $\frac{\partial u(f, T)}{\partial f} = 0$, using the high frequency approximation to $u(f, T)$, gives $\frac{hf}{k_B T} = \frac{4}{3}$. Find

the wavelength, λ , of the peak by this equation at $T = 5400K$.

(d) Finally, we can try the *low frequency approximation*, $u(f, T) = \frac{8\pi hf^2}{c^3} k_B T$, and find $\frac{\partial u(f, T)}{\partial f} = 0$. Find

the wavelength, λ , of the peak by this equation at $T = 5400K$.

(e) Visible light runs from 400 nm to 750 nm, with a peak sensitivity in the human eye at 500 nm. Sketch this function on this graph, as well as the peaks from (a), (b), (c) and (d).

Planck's Blackbody Equation (for fixed Temp)

