PHYS-309(2) (Kaldon-21722) **Name** ____
WMU - Winter 2000
Exam 000 - 000,000 points **Book Title** ____

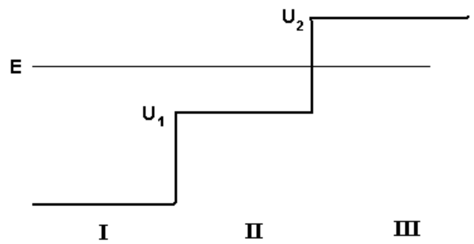
6/11/1999•Rev.1m1

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions

For all problems, use $c = 2.998 \times 10^8$ m/s; $h = 6.626 \times 10^{-34}$ J·s; $\hbar = 1.055 \times 10^{-34}$ J·s.

The Wall (25,000 points)

1.) Consider a plane wave in Region I heading toward the wall in the diagram shown. The energy E is in between U_1 and U_2. From the forms of ψ (x) and ϕ (t) given as potential solutions, write up the *form* of the solutions in Regions I, II and III. Then indicate what your B.C. (boundary conditions) would be between Regions. *Do not attempt to solve anything!*



Note: The original sketch was done by hand – but I believe this is what the picture looked like – Dr. Phil - 15 March 2004

Work, Work, Work (25,000 points)

2.) Electrons in *excited states*, those not in the ground state electronic configuration, want to get to lower energy states if they are available. (a) In the simple classically derived Bohr atom, where n is the only quantum number, all transitions are possible. List all the transitions to take an electron in the $n = 5$ state to the $n = 1$ state. *Example:* $5 \to 1$ and $5 \to 2 \to 1$. How many different paths is this?

(b) For the longest of the paths you found in (a), count up the number of degeneracies of spin-up electrons in each state. Multiplied together, this finds the total number of different combinations going from $n = 5$ state to the $n = 1$ state for a particular pathway.

(c) For the QM hydrogen atom, not all paths are possible. There are selection rules that say that $\Delta l = \pm 1$ in each step. Thus a 5s electron can't directly jump to the 1s state. There are only five allowed transitions. Use the number of degeneracies in spin-up electrons for each state and find the number of combinations for the first path listed here.

1.) 5s → 4p → 3d → 2p → 1s [1]
2.) 5s → 4p → 3s → 2p → 1s
3.) 5s → 4p → 1s
4.) 5s → 3p → 1s
5.) 5s → 2p → 1s

(d) In the real atom, there are energy differences between the levels. 4p and 4s don't have the same energy. There are now seven allowed transitions. Use the number of degeneracies in spin-up electrons for each state and find the number of combinations for the first path listed here.

1.) 5s → 4p → 4s → 3p → 3s → 2p → 1s [2]
2.) 5s → 4p → 4s → 3p → 1s [3]
3.) 5s → 4p → 4s → 2p → 1s [4]
4.) 5s → 4p → 3s → 2p → 1s
5.) 5s → 4p → 1s
6.) 5s → 3p → 3s → 2p → 1s [5]
7.) 5s → 3p → 1s

[1] This transition pathway is unique to this problem, because the 3d state is higher than the 4s state in the shifted system.
[2] This transition pathway is unique, because the s and p states are not equal in the shifted system.
[3] This transition pathway is unique, because the s and p states are not equal in the shifted system.
[4] This transition pathway is unique, because the s and p states are not equal in the shifted system.
[5] This transition pathway is unique, because the s and p states are not equal in the shifted system.

Bohring Atoms (25,000 points)

3.) (a) Write down the ground state electronic configuration ($1s^2$…) for *iodine*.

(b) What are the four quantum numbers for the *next* electron that would be added?

Below are the functional forms of the hydrogenic atom solutions through $n = 3$.

n	l	m_l	Eigenfunction
1	0	0	$A_{100}e^{-Zr/a_0}$
2	0	0	$A_{200}\left(2-\frac{Zr}{a_0}\right)e^{-Zr/2a_0}$
2	1	0	$A_{210}\left(\frac{Zr}{a_0}\right)e^{-Zr/2a_0}\cos\theta$
2	1	±1	$A_{21\pm1}\left(\frac{Zr}{a_0}\right)e^{-Zr/2a_0}\sin\theta e^{\pm i\varphi}$
3	0	0	$A_{300}\left(27-18\frac{Zr}{a_0}+2\frac{Z^2r^2}{a_0^2}\right)e^{-Zr/3a_0}$
3	1	0	$A_{310}\left(6-\frac{Zr}{a_0}\right)\frac{Zr}{a_0}e^{-Zr/3a_0}\cos\theta$
3	1	±1	$A_{31\pm1}\left(6-\frac{Zr}{a_0}\right)\frac{Zr}{a_0}e^{-Zr/3a_0}\sin\theta e^{\pm i\varphi}$
3	2	0	$A_{320}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}(3\cos^2\theta-1)$
3	2	±1	$A_{32\pm1}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}\sin\theta\cos\theta e^{\pm i\varphi}$
3	2	±2	$A_{32\pm2}\frac{Z^2r^2}{a_0^2}e^{-Zr/3a_0}\sin^2\theta e^{\pm 2i\varphi}$

Source: Quantum Physics of Atoms, Molecules, Nuclei and Particles / Robert Eisberg and Robert Resnick. New York: John Wiley & Sons, 1974. *aka* "The Silver Bullet".

(c) Show, in any reasonable argument or fashion, that it is not possible to construct the $n = 3$ function from linear combinations of $n = 1$ and $n = 2$ functions.

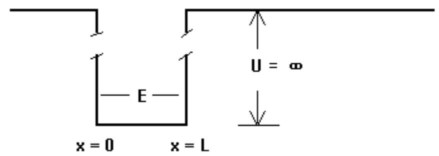
(d) From the structure of the functions for $n = 1, 2, 3$, construct the form for function $n = 8$, $l = 7$, $m_l = \pm 7$, $Z = 17$. *Do not worry about the value of constants like 27 & 18 – we are concerned only with the terms in* r, θ, φ, *and any quantum numbers like* n, l, m_l.

(e) What electron orbital is this?

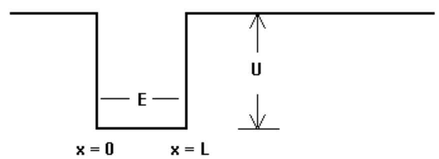
"O Schrödinger, Schrödinger, Wherefore Art Thou?" she psi'ed… (25,000 points)

4.) (a) Why do we have to square ψ^2 to see the probability, when ψ describes the whole wavefunction?

(b) For the infinite square well problem in 1-D, why did we use ψ = `A sin(kx)`, when the general solution to the Schrödinger equation is ψ = `A sin(kx) + B cos(kx)`?



(c) For the finite square well problem in 1-D, where E < U, why do we use the general solution ψ = `A sin(kx) + B cos(kx)` for the particle in the well, when we used just ψ = `A sin(kx)` for the infinite square well problem?



(d) Explain why we have to normalize the solution ψ.

(e) If the Schrödinger Equation is a wave equation, then what is waving?