

State Any Assumptions You Need To Make -- Show All Work -- Circle Any Final Answers

You've Got To Believe in Magic... (40,000 points)

1.) (a) The magic numbers are 2, 8, 20, 28, 50, 82, 126. There are no stable isotopes of Technetium (Tc) or Promethium (Pm). Take a look at the known isotopes of Tc and Pm in *Appendix B: Table of Atomic Masses*, and analyze Z and N in terms of the magic numbers. For what atomic number A might you expect there to be magic numbers for N in Tc and Pm? How "far" is this from the known isotopes, in terms of added or subtracted neutrons? Why do you think that Tc and Pm are unstable?

(b) Compare the number of stable isotopes that have N=20 with those that have N= 21 and N=22.

I Keep Telling Myself: I am Perfectly Normal, I am Perfectly Normal... (40,000 points)

2.) $\psi_{100} = A_{100}e^{-Zr/a_0}$ is the Schrödinger eigenfunction for the hydrogenic 1s orbital. (a) What system are we talking about if $Z = 3$?

(b) Normalize the wavefunction by integrating to find A_{100} for $Z = 3$. *Hint: There is no angular dependence for s-orbitals, so the angular part of the integration just gives you 4π .*

Give me a "T"! Give me a "c"! Whadya Get? Technetium! (40,000 points)

3.)(a) Although there are no stable isotopes of Technetium (Tc), several of the isotopes have half-lives that can be measured in the millions of years. Pick one of the three longest living isotopes of Tc. If you had 1 mole of this isotope four billion years ago, how many atoms should have survived to today? *Note: If you get a number less than one, report the fraction, not zero, even though you know you won't get, say, half an atom.*

(b) For your isotope of Tc, indicate what products you would get via the following decay modes: α emission, β^+ emission, β^- emission and electron capture.

(c) Check in *Appendix B*, and see which of your answers in (b) are listed. If they are, give their half-life.

(d) If you had an atom of Tc and it emitted a γ -ray, what product do you get? What could you say about the atom, or more specifically the nucleus, before the emission?

The Hit Rock Group: Smashing Particles! (40,000 points)

4.(a) How big a mass in kg is $1 \text{ MeV}/c^2$? (*Don't look it up – do the math.*)

(b) An imaginary particle called a kald-on (rest mass of the $k = 300 \text{ MeV}/c^2$) is sitting at rest when it spontaneously decays into two wo-ons¹ (rest mass of the $w = 100 \text{ MeV}/c^2$). The wo-ons could be identical, or one could be a wo-on and one could be an anti-wo-on, or one could be charged and the other neutral. Suggest some likely combinations of wo-ons if the kald-on has a charge of $q=0$, $q=+e$ or $q=+2e$.

(c) Find the speed of the two wo-ons. Assume that the particles are relativistic if the classical speed is more than $0.4 c$, and use the relativistic equations.

¹ Phil Kaldon and Dave Woon shared an office in the Physics Department at Michigan Tech as grad students. Sue Hill, another graduate student, came up with the particle names.

“An Electron and a Positron Walked Into a Bar, See...” (40,000 points)

5.(a) An electron and a positron are heading toward each other, each traveling at half the speed of light. They will collide in 2.0 ms in the lab frame. Does the electron see the positron approach at the speed of light? Why or why not?

(b) How long do the particles think that they have before the collision? How far away does the electron think the positron is, in the electron's rest frame?

(c) What are the energies of the two γ -rays that come from the annihilation of the electron-positron pair?

(d) Use the relativistic momentum to find the deBroglie wavelength, λ , and the Heisenberg uncertainty of the position, Δx , of the positron. Do you think that the positron (or the electron for that matter) are more particle-like or more wavelike in this problem?