A Modern Physics Smorgasbord (50,000 points)

1.) (a) Compton himself measured the scattering of X-rays off the electrons in graphite. For a scattering angle $\theta = 135°$, and incoming X-rays with a wavelength $\lambda_0 = 0.0711\ nm$, find the wavelength λ of the scattered X-rays.

$$\lambda - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$$
$$\lambda = \lambda_0 + \frac{h}{m_e c}(1-\cos\theta)$$
$$= 0.0711 nm + \frac{6.63\times10^{-34}\,J\cdot s}{(9.11\times10^{-31}\,kg)(3.00\times10^{8}\,m/s)}(1-\cos 135°)$$
$$= 7.11\times10^{-11}m + (2.426\times10^{-12}m)(1.707) = 7.5241\times10^{-11}m$$
$$= 0.07524 nm$$

(b) Ordinary gold (Z=79) has 118 neutrons. A one-pound gold bar (0.454 kg = 454 grams) contains approximately how many atoms of gold?

$$Z = 79, N = 118,\ \ A = Z + N = 79 + 118 = 197$$
$$1\ mole\ is\ 197\,grams$$
$$n = \frac{454\,grams}{197\,grams/mole} = 2.305\,moles$$
$$N = nN_A = (2.305)(6.02\times10^{23}) = 1.388\times10^{24}$$

(c) A 1971 Cadillac Sedan de Ville, mass = 2250 kg, is driving along with a speed of 68 mph ($v = 30.4\ m/s$). Find the de Broglie wavelength of this big block V-8 carburetor powered hunk of iron.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J\cdot s}{(2250kg)(30.4m/s)} = 9.693\times10^{-39}\,m$$

(d) The kinetic energy of the ground state electron (n=1) in hydrogen is 13.6 eV. Find the de Broglie wavelength of this electron.

$$K = \tfrac{1}{2}mv^2\ \ OR\ \ K = \frac{p^2}{2m}$$
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(13.6eV)(1.60\times10^{-19}\,J/eV)}{(9.11\times10^{-31}\,kg)}} = 2.186\times10^{6}\,m/s$$
$$OR\ \ \ p = \sqrt{2Km} = \sqrt{2(13.6eV)(1.60\times10^{-19}\,J/eV)(9.11\times10^{-31}\,kg)}$$
$$= 1.991\times10^{-24}\,kg\cdot m/s$$
$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\,J\cdot s}{(1.991\times10^{-24}\,kg\cdot m/s)} = 3.330\times10^{-10}m$$
$$OR$$
$$= \frac{h}{mv} = \frac{6.63\times10^{-34}\,J\cdot s}{(9.11\times10^{-31}\,kg)(2.186\times10^{6}\,m/s)} = 3.330\times10^{-10}m$$

(c) The rest mass energy of an electron is 0.511 MeV (0.511×10^6 eV). In "pair creation", a 1.022 MeV or higher energy gamma ray can create a pair of particles (electron e^- and positron e^+), with any energy over the total rest mass energy of 1.022 MeV going into kinetic energy – this reaction conserves momentum, mass-energy and charge. It is thought that such a process can briefly and spontaneously occur in vacuum, by "borrowing" 1.022 MeV for a short time and then letting the electron-positron pair fall back together and annihilate each other before Nature notices that the energy is "missing". Use the Heisenberg Uncertainty Relation to find the maximum time this event can take without anyone able to detect such a thing has occurred. Let ΔE =1.022MeV.

$$\Delta E\ \Delta t \geq \frac{\hbar}{2}$$
$$\Delta t \geq \frac{\hbar}{2\Delta E} = \frac{(6.58\times10^{-16}\,eV\cdot s)}{2(1.022\times10^{6}\,eV)} = 3.219\times10^{-22}\,s$$
$$OR\ \ = \frac{(1.05\times10^{-34}\,J\cdot s)}{2(1.022\times10^{6}\,eV)(1.60\times10^{-19}\,J/eV)} = 3.211\times10^{-22}\,s$$

So if the time is less than 3.21×10^{-22} seconds, you can temporarily cheat conservation of energy.
(But you have to remember to put it back, Cinderella!)

"*All That Glitters is Not Crystalline Carbon*" (50,000 points) ‡

2.) Carbon (Z = 6) comes in several forms, one of which is graphite – carbon atoms are arranged in flat sheets which then stack on top of each other and slide around. The carbon-carbon bond distance – the distance between carbon atoms is much shorter for within the sheets (1.42 Å) than between the sheets (3.41 Å). In diamond, another form of solid carbon, the bond length is 1.54 Å. (a) For a first approximation of the carbon-carbon distance, we note that in a multi-electron carbon atom, the bonding electrons are in the n = 2 orbits. So let us calculate r_2 for the n = 2 orbital in the hydrogenic ion, C^{5+}, then multiply by 2 to get the distance between nuclei. Which of the above bond lengths is your calculation closest to? *1 Å = 0.1 nm =* 1×10^{-10} *m*

$$r_n = \frac{n^2}{Z}a_0$$
$$r_2 = \frac{2^2}{6}\left(0.529\times10^{-10}m\right) = 3.527\times10^{-11}m$$
$$d = 2r_2 = 2\left(3.527\times10^{-11}m\right) = 0.7053\times10^{-10}m = 0.7053\text{Å}$$

What a surprise – not a good match to any of these.
(Guess those other five electrons actually DO something!)

(b) According to <u>The Handbook of Physics and Chemistry</u>, the photoelectric work function for carbon is 4.81 eV. We can try to approximate this by again using the hydrogenic carbon ion C^{5+} and finding the energy E_2 of the electron from the n=2 orbital. (If the n=2 electron absorbed a photon of energy $|E_2|$, it would be free of the carbon ion.) Compare the energy you calculate with the work function for carbon.

$$E_n = -\frac{Z^2}{n^2}2.18\times10^{-18}J = -\frac{Z^2}{n^2}13.6eV$$
$$E_2 = -\frac{6^2}{2^2}13.6eV = -122.4eV$$

Gee, strike two. Another bad match.
(We shouldn't be surprised – why would you use a hydrogenic carbon ion to simulate multi-electron carbon atoms in graphite?)

‡ NOTE: Several of these calculations involve trying to find real numbers using hydrogenic ions. Is this a good idea?

(c) Conversely, we could take the energy 4.81 eV and plug into the hydrogenic carbon equation and calculate the *n* which has the energy, E_n .

$$E_n = -\frac{Z^2}{n^2}13.6eV$$
$$4.81eV = \frac{6^2}{n^2}13.6eV$$
$$n^2 = \frac{6^2}{4.81eV}13.6eV$$
$$n = \sqrt{\frac{36}{4.81eV}13.6eV} = 10.09 = 10$$

Consider that in a metal, the conduction electrons are roaming around, not connected to any one atom. I'd say that such a high value for *n* means we're not really connected to one atom.

(d) If we want to drive an electron from a real carbon surface with a Kinetic Energy of up to 15.0 eV, find the energy *E*, frequency *f* and wavelength *λ* of the photon required.

$$K_{\max} = hf - \phi$$
$$E = hf = K_{\max} + \phi = 15.0eV + 4.81eV = 19.81eV$$
$$f = \frac{E}{h} = \frac{19.81eV}{4.14\times10^{-15}eV\cdot s} = \frac{19.81eV\left(1.60\times10^{-19}J/eV\right)}{6.63\times10^{-34}J\cdot s}$$
$$= 4.785\times10^{15}Hz \;\; or \;\; 4.781\times10^{15}Hz$$
$$c = f\,\lambda$$
$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}m/s}{4.785\times10^{15}Hz} = 6.270\times10^{-8}m$$

(e) With the distance between layers of graphite being 3.41 Å, find the wavelength *λ* of the light waves scattered by the *n = 1* Bragg reflection for an angle *θ = 45°*.

$$2d\sin\theta = n\lambda$$
$$\lambda = 2d\sin\theta = 2\left(3.41\text{Å}\right)\left(\sin45^\circ\right) = 4.822\text{Å}$$