XF.3a **309**

PHYS-309 (3) (Kaldon-21722) **Name** S O L U T I O N

WMU - Spring 2004

Final Exam - 200,000 points

Rev. 04/20/2004 Tu.7

State Any Assumptions You Need To Make – Show All Work – Circle Any Final Answers
Use Your Time Wisely – Work on What You Can – Be Sure to Write Down Equations
Feel Free to Ask Any Questions

FINAL EXAM [FORM - A]

PHYS-309 (KALDON-3)

SPRING 2004

WMU

Magic Numbers: 2, 8, 20, 28, 50, 82, 126

$c = 3.00 \times 10^{8}$ m/s

$a_0 = a_B = 0.529$ Å $= 0.529 \times 10^{-10}$ m

$E_0 = E_R = 13.6$ eV

$e = 1.60 \times 10^{-19}$ C

1 eV $= 1.60 \times 10^{-19}$ J

$m_e = 9.11 \times 10^{-31}$ kg

$h = 6.63 \times 10^{-34}$ J·s

$\hbar = 1.05 \times 10^{-34}$ J·s

$m_p = m_n = 1.67 \times 10^{-27}$ kg

"We may not always know what we're doing, but I'm certain about the Uncertainty Principle."

Y in the World Did This Happen? (50,000 points)

1.) Ytterby, Sweden, must have some weird dirt, contributing four tongue twisting elements – Yttrium, Terbium, Erbium and Ytterbium to science. (a) Element 39, Yttrium, has only one stable isotope, ${}^{89}_{39}Y$. Why could you guess that this is, at least, the most *common* isotope. *Short answer, please!*

39 Y Yttrium 88.9059

Rounding off the atomic weight from the Periodic Table usually gives the most common isotope.
88.9059 → 89

See answer to (b) regarding Magic Numbers…

(b) Why should Yttrium-89 be so stable? *Short answer!*

N = A - Z = 89 - 39 = 50 50 is a Magic Number and Magic Numbers are "Good".

(c) Terbium-158 decays by both β^+ and β^-. Terbium-157 decays by e.c. (electron capture). Remarkably, all three decays have a half-life of about 150 years. Write down balanced nuclear reactions for all three decays.

65 Tb Terbium 158.925

$${}^{158}_{65}Tb \rightarrow {}^{158}_{64}Gd + {}^{0}_{1}e + {}^{0}_{0}\nu \qquad (\beta^+)$$

$${}^{158}_{65}Tb \rightarrow {}^{158}_{66}Dy + {}^{0}_{-1}e + {}^{0}_{0}\overline{\nu} \qquad (\beta^-)$$

$${}^{157}_{65}Tb + {}^{0}_{-1}e \rightarrow {}^{157}_{64}Gd + {}^{0}_{0}\nu \qquad (\text{e.c.})$$

(d) Erbium's atomic weight of 167.26 is a weighted average of all its stable isotopes. Why might all but one of the stable isotopes have an even-numbered value for A? *Short answer.*

68	Er	167.26	Erbium
	162	161.928 774	0.14%
	164	163.929 198	1.61%
	166	165.930 290	33.6%
	167	166.932 046	22.95%
	168	167.932 368	26.8%
	169	168.934 588	9.40 d (β^-)
	170	169.935 461	14.9%
	172	171.939 353	49.3 h (β^-)

68 Er Erbium 167.26

Z = 68 is Even
Even-Even nuclei are very stable,
Even-Odd and Odd-Evens are less,
Odd-Odd nuclei are far less likely.
So if Z & N are Even, then A is Even.

(e) Ytterbium has 70 electrons. Write down the electron structure of the ytterbium ground state. *You can speed the process by starting with the symbol [Xe] and only adding the electrons past xenon.*

70 Yb Ytterbium 173.04

[Xe] $6s^2\,4f^{14}$ or $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}\,4p^6\,5s^2\,4d^{10}\,5p^6\,6s^2\,4f^{14}$

Helium – First Discovered During a Solar Eclipse in 1868 (50,000 points)

2.) (a) Helium is so anti-social chemically/electronically, it does not want to enter into any sort of ordinary chemical reaction. Helium doesn't even want to stick to another helium, making it pretty much a perfect candidate for being an Ideal Gas. The Ideal Gas Law is $P\,V = n\,R\,T$. If the pressure P is kept at 1 atmosphere and the number of moles, n , is held constant, then it looks like at absolute zero (T = 0 K) the volume V goes to zero. So why doesn't the helium disappear at absolute zero at 1 atmosphere pressure? *Short answer*.

(1) Helium isn't a gas at absolute zero at 1 atmosphere pressure (it's a liquid below 4.2 K).

(2) Zero point energy – recall that ground states are not zero energy or zero width in size, so Quantum Mechanics prevents the classical Ideal Gas Law from making *He* disappear.

(b) Find the radius r_1 of a hydrogenic He^+ helium ion.

$$r_n = \frac{n^2}{Z} a_0 \;\; ; \;\; r_1 = \frac{1^2}{2}\left(5.29\times10^{-11} m\right) = 2.645\times10^{-11} m$$

(c) Liquid helium has a mass-to-volume ratio (density) of $\rho = 124.96\ kg/m^3$. The atomic mass of helium is 4.0026. That means every 4.0026 grams of helium (0.0040026 kg) is one mole of helium, 6.02×10^{23} atoms. How many atoms of helium are in 124.96 kg of liquid helium?

$$n = \frac{124.96 kg}{0.0040026 kg / mole} = 31{,}220 moles$$
$$N = n\,N_A = (31{,}220)\left(6.02\times10^{23}\right) = 1.879\times10^{28}$$

(d) Find the approximate volume of a single helium atom in liquid helium. *Hint: We have 1.00 m³ of liquid He.*

$$\frac{1.00 m^3}{1.879\times10^{28}} = 5.322\times10^{-29} m^3$$

(e) The hydrogenic helium ion does not have the same size as a neutral helium atom. But we can estimate the volume of a helium ion by taking the result from (b), and plugging it into $V = \frac{4}{3}\pi r^3$. Compare this volume to the answer in (d) and also the volume using the textbook radius for the helium atom of r = 0.05 nm.

from hydrogenic helium ion

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi\left(2.645\times10^{-11} m\right)^3 = 7.751\times10^{-31} m^3$$

from textbook radius

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi\left(0.05\times10^{-9} m\right)^3 = 5.236\times10^{-31} m^3$$

from (d), the radius appears to be a very loose 2.33×10^{-10} m (very unsticky!)

A Relative Family Portrait (50,000 points)

3.) About a decade ago, NASA proposed sending a probe called TAU up above the Solar System so it could look back and take photographs of the whole Solar System at once. Kind of like stepping back with your camera so you can include all your family in the picture. Only this time, TAU stands for a Thousand Astronomical Units. An A.U. is the average distance from the Earth to the Sun, which is 93,000,000 miles or 149,600,000 km. And we want to send a probe 1000. times further than this. (a) From 1000. A.U. out, how long would it take for the radio to send a picture back to Earth?

$$d = v\,t \;\; ; \; d = (1000)(149{,}600{,}000{,}000 m) = 1.496\times10^{14} m$$
$$t = \frac{d}{v} = \frac{1.496\times10^{14} m}{3.00\times10^{8} m/s} = 498{,}700 \sec \; (= 5.772\, days)$$

(b) One problem is that at typical rocket speeds, it would take some 300. years to travel that far. Find the average speed of the TAU probe on its way to its parking space in the sky.

$$t = (300.\,years)(365 days/year)(24 hours/day)(3600 \sec/hour) = 9.461\times10^{9} \sec$$
$$v = \frac{d}{t} = \frac{1.496\times10^{14} m}{9.461\times10^{9} \sec} = 15{,}810 m/s \;\; (= 35{,}370 mph)$$

(c) If we had a rocket that could travel at 80.0% the speed of light, 0.800 c , how long (time) would it take to make the same trip, as measured from the Earth?

$$d = v\,t \;\; ; \; v = 0.800c = (0.800)\left(3.00\times10^{8} m/s\right) = 2.400\times10^{8} m/s$$
$$t = \frac{d}{v} = \frac{1.496\times10^{14} m}{2.400\times10^{8} m/s} = 623{,}300 \sec \; (= 7.214\, days)$$

(d) How far (distance) would the relativistic TAU probe say it has to travel and how long (time) would it say the trip took?

$$\beta = 0.800 \ ; \ L_0 = 1.496\times10^{14}\,m$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.800)^2}} = 1.667$$
$$L = \frac{L_0}{\gamma} = \frac{1.496\times10^{14}\,m}{1.667} = 8.974\times10^{13}\,m$$
$$t = \frac{d}{v} = \frac{8.974\times10^{13}\,m}{2.400\times10^{8}\,m/s} = 373{,}900\sec$$
$$OR \quad t' = \gamma\, t_0$$
$$t_0 = \frac{t'}{\gamma} = \frac{623{,}300\sec}{1.667} = 373{,}200\sec$$

(e) According to a calculation we did in class, Dr. Phil suggests that one needs to take account of relativity when $v > 0.416\,c$, at the "ten percent level" in measurement differences – or $\gamma = 1.10$. Find the speed v which requires relativity at the "one percent level" in measurement difference – or $\gamma = 1.01$.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\sqrt{1-\beta^2} = \frac{1}{\gamma}$$
$$1-\beta^2 = \frac{1}{\gamma^2}$$
$$\beta^2 = 1-\frac{1}{\gamma^2}$$
$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.01)^2}} = 0.1404 \ or \ 14.04\%$$
$$v = \beta c = 0.1404c = (0.1404)(3.00\times10^{8}\,m/s)$$
$$= 42{,}120{,}000\,m/s = 4.212\times10^{7}\,m/s$$

Relativistic Mass Energy Conversion Wave Uncertainties Abound Here (50,000 points)

4.) (a) Our very brief look at particle physics included the accelerator reaction $p + p \rightarrow p + p + p + \overline{p}$, or one proton is accelerated to high speed and runs into a proton at rest. The collision results in creating a proton-anti-proton pair from the conversion of excess kinetic energy into mass. The energy to create either a proton or an anti-proton is $E = m_p c^2$ or 938.3 MeV. Find the minimum speed v of the moving proton to create the proton-anti-proton pair. *Give the answer in terms of β, the fraction of the speed of light.*

$$K = 2(938.3\,MeV) = 1877\,MeV(1.60\times10^{-13}\,J/MeV)$$
$$= 3.00\times10^{-10}\,J$$
$$K = (\gamma-1)m_0c$$
$$\gamma - 1 = \frac{K}{m_0c} \ ; \ \gamma = \frac{K}{m_0c} + 1 = \frac{2(938.3\,MeV)}{(938.3\,MeV)} + 1 = 2 + 1 = 3.00$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \ ; \ \sqrt{1-\beta^2} = \frac{1}{\gamma} \ ; \ 1-\beta^2 = \frac{1}{\gamma^2} \ ; \ \beta^2 = 1-\frac{1}{\gamma^2}$$
$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(3.00)^2}} = 0.9428 \ or \ 94.28\% \text{ the speed of light}$$

(b) Find the de Broglie wavelength of a different proton traveling at 100. m/s.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\,J\cdot s}{(1.67\times10^{-27}\,kg)(100.\,m/s)} = 3.970\times10^{-9}\,m$$

(c) If we know the uncertainty of the proton's speed in part (b) to within $\Delta v = 0.100\ m/s$, find the uncertainty in the proton's position, Δx , according to the Heisenberg Uncertainty Principle.

$$\Delta x\,\Delta p_x \geq \frac{\hbar}{2}$$
$$\Delta x \geq \frac{\hbar}{2\Delta p_x} = \frac{1.05\times10^{-34}\,J\cdot s}{2\left((1.67\times10^{-27}\,kg)(0.100\,m/s)\right)}$$
$$\geq 3.144\times10^{-7}\,m$$

(d) Suppose the proton in (b) is confined to a box of width *a*, where the de Broglie wavelength from (b) is the fundamental, or n = 1 state, of this particle-in-a-box. How big is *a* ? *Hint: Think simple!*

Fundamental is half a wavelength

$$a = \frac{\lambda}{2} = \frac{3.970 \times 10^{-9}\,m}{2} = 1.985 \times 10^{-9}\,m$$

(e) For the last question, let's consider sodium (Na), simply because the numbers are handy. The ionization energy (or first ionization potential) of Na is 5.138 eV (p.328 Serway M&M). The photoelectric effect work function for Na is 2.28 eV (p. 74). Define what each energy represents and give a plausible physics explanation as to why the work function is so much less than the ionization energy, when both involve removing an electron.

The ionization energy of 5.138 eV is the energy to remove one electron from one Na atom.

The work function of 2.28 eV is the minimum energy of the incident photon that can eject one electron from the surface of Na metal.

Has to do with the metal conduction band electrons, at least one per atom, which belong to the metal rather than any one particular atom. That means they are less strongly bound, because they can be farther from the nuclear centers and are screened more by the inner electrons. So it is easier to remove an electron in the photoelectric effect than to ionize it, according to the data.