ENGR 1990 Engineering Mathematics
Homework #10 – Integrals

1. A hardening spring has the force-displacement function \( f(x) = 100 + 10x + x^2 \) (lb). The work done by the spring as it is stretched over some displacement interval is the negative of the integral of the force-displacement function over that interval. Estimate the integral and the work done by the spring as it is stretched from \( x = 0 \) to \( x = 2.5 \) inches by breaking the area into a sequence of trapezoids of width \( \Delta x = 0.5 \) (in).

\[
\begin{array}{|c|c|c|c|}
\hline
x & f(x) & Interval & f_{avg} & \Delta x \\
\hline
0 & -- & & & \\
0.5 & 1 & 1 & 0.5 \\
1 & 2 & 2 & 0.5 \\
1.5 & 3 & 3 & 0.5 \\
2 & 4 & 4 & 0.5 \\
2.5 & 5 & 5 & 0.5 \\
\hline
\end{array}
\]

\[ W_{A \to B} = -\int_{x_a}^{x_b} f(x) \, dx \]

2. Using the same spring force-displacement function given in problem (1), find the work done by the spring using anti-derivatives. Calculate the percent error of the estimate found in problem (1).

3. A car has the acceleration profile shown, and its initial position and velocity are zero. Given that

\[
v(t) = \int a(t) \, dt \quad \text{and} \quad s(t) = \int v(t) \, dt,
\]

find (a) the velocity function \( v(t) \), (b) the displacement function \( s(t) \), and (c) the total distance traveled by the car for \( 0 \leq t \leq 20 \) (sec). Sketch the functions.

4. A ball that is thrown upward has velocity \( v(t) = 75 - 32.2t \) (ft/s). Given that the displacement function of the ball is

\[
s(t) = \int v(t) \, dt,
\]

find (a) the displacement of the ball from its original position after 3.5 seconds, and (b) the total distance traveled by the ball during the 3.5 second period.

5. A ball is thrown upward with an initial velocity of \( v_0 = 20 \) (m/s) from an initial height of \( y_0 = 8 \) (m) and has a constant downward acceleration of \( a_0 = -9.81 \) (m/s^2). Given that \( v(t) = \int a(t) \, dt \) and \( y(t) = \int v(t) \, dt \), find (a) the velocity function \( v(t) \), and (b) the position function \( y(t) \).
6. The simply supported beam has a uniformly distributed load over the left half of the beam. For a beam of length \( L = 10 \text{ (ft)} \) and a load \( \omega = 100 \text{ (lb/ft)} \), the internal shearing force is

\[
V(x) = \begin{cases} 
375 - 100x \text{ (lb)} & 0 \leq x \leq 5 \text{ (ft)} \\
-125 \text{ (lb)} & 5 \leq x \leq 10 \text{ (ft)} 
\end{cases}
\]

Given that the internal bending moments at \( A \) and \( B \) are zero, find \( M(x) = \int V(x) \, dx \) the internal bending moment as a function of \( x \).

7. A current \( i(t) = 2e^{-2t} \) (amps) is applied to a capacitor with capacitance \( C = 0.25 \text{ (f)} \). Given that \( v(t) = \frac{1}{C} \int i(t) \, dt \), find the

a) voltage \( v(t) \). Assume \( v(0) = 0 \).

b) power, \( p(t) = v(t) \cdot i(t) \).

c) total energy, \( W(t) = \int_0^t p(t) \, dt \) (joules).

What is the energy at \( t = 1, 2, 3, 4 \) (sec)? What is the limit of the energy as \( t \to \infty \)?

8. A voltage \( v(t) = 10 \sin(120\pi t) \) (volts) is applied to an inductor with inductance \( L = 250 \text{ (mh)} \). Find the current \( i(t) \), given that \( i(t) = \frac{1}{L} \int v(t) \, dt \). Assume \( i(0) = 0 \).