1. A ball is thrown off a tower at a height of 60 (ft) at a speed of 50 (ft/s) as shown. The \( X \) and \( Y \) positions of the ball are given as functions of time.

\[
x(t) = 30t \quad \text{(ft)} \quad y(t) = 60 + 40t - 16.1t^2 \quad \text{(ft)}
\]

Using the table of derivatives and the rules for differentiation, find

(a) \( \dot{x}(t) \) and \( \ddot{x}(t) \) the first and second time derivatives of \( x(t) \)
(b) \( \dot{y}(t) \) and \( \ddot{y}(t) \) the first and second time derivatives of \( y(t) \)
(c) the slope of \( y(t) \) when \( t = 2 \) (sec)
(d) the time \( t^* \) when the ball reaches its maximum height
(e) the \( X \) and \( Y \) coordinates of the ball when it reaches its maximum height
(f) \( \mathbf{V} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \) the velocity vector of the ball at \( t = 2 \) (sec)
(g) \( y(x), \frac{dy}{dx}(x), y''(x) = \frac{d^2y}{dx^2}(x) \)
(h) using the results of part (g), find the \( X \) coordinate of the ball when it reaches its maximum height
(i) find the equation of the line that is tangent to \( y(x) \) at \( x = 80 \) (ft)

2. A spring-mass system with \( m = 2 \) (slugs), \( k = 72 \) (lb/ft), and no damping is shown in the diagram. The system is given an initial displacement of \( x_0 = 1 \) (ft) and initial velocity of \( v_0 = 9 \) (ft/s).

a) Find \( x(t) \) as a sum of sine and cosine functions.
b) Find \( x(t) \) as a single sine function with magnitude and phase.
c) Find the time when the mass first reaches its largest displacement.
d) Find \( T \) the period of the oscillation.
e) Find \( v(t) = \frac{dx}{dt} \) the velocity of the mass as a single sine function.
f) Find \( a(t) = \frac{dv}{dt} \) the acceleration of the mass as a single sine function.
g) Find the first time when the velocity \( v(t) \) is maximum or minimum. Is it a maximum or minimum?
3. The response of an over-damped, spring-mass-damper system is

\[ x(t) = 3e^{-5t} - 2e^{-20t} \text{ (ft)} \]

Using the table of derivatives and the rules for differentiation, find

a) \( v(t) = \dot{x}(t) \) the velocity function
b) \( a(t) = \ddot{v}(t) = \ddot{x}(t) \) the acceleration function
c) \( x(0), v(0), a(0) \) the initial position, velocity and acceleration of \( m \)
d) the time \( t^* \) when \( m \) has its maximum downward displacement