ENGR 1990 Engineering Mathematics
Lab/Recitation #14 – Review for Final Exam

1. a) A force \( F \) has a magnitude of 500 (lb) and acts at an angle of \( \theta = 120 \) (deg) to the positive \( X \)-axis. Express \( F \) in terms of the unit vectors \( i \) and \( j \).

b) A unit vector \( n \) is pointed along a line that passes through the points \( A \) and \( B \). The \( XY \) coordinates of the two points are \( A(2,3) \) inches and \( B(7,15) \) inches. Express \( n \) in terms of the unit vectors \( i \) and \( j \).

c) Find \( F_\parallel \) the component of \( F \) that is parallel to \( n \).

2. a) A force \( F \) has a magnitude of 500 (lb) and acts at an angle of \( \theta = 120 \) (deg) to the positive \( X \)-axis. The force is located at a point \( A \) with coordinates \( (5,2) \) inches. Find \( M \), the moment of the force about the origin \( (0,0) \).

b) Find \( d \) the perpendicular distance from the line of action of \( F \) to the origin \( (0,0) \).

3. A weight of 500 (lb) is supported by the two wires as shown. By setting the sum of the forces acting at point \( A \) to zero \( (\sum F = 0) \), complete the following.

a) Find the two equations that must be solved to find the magnitudes of the forces \( F_1 \) and \( F_2 \).

b) Solve the two simultaneous equations using substitution and Cramer’s rule.

4. A voltage \( v(t) = 220 \cos(120\pi t) \) volts is applied to the RLC series circuit with \( R = 100 \) \( \Omega \), \( C = 50 \mu F \), and \( L = 500 \) mh. Given that the total impedance is \( Z = Z_R + Z_C + Z_L \), find

a) \( Z \) in both rectangular and polar form

b) \( I \) the complex current in both rectangular and polar form

c) \( i(t) \) the current as a function of time

5. A voltage \( v(t) = 110 \cos(120\pi t) \) volts is applied to the RLC parallel circuit with \( R_1 = R_2 = 100 \) \( \Omega \), \( C = 25 \mu F \), and \( L = 500 \) mh. Given that \( Z_1 = Z_{R_1} + Z_C \) and \( Z_2 = Z_{R_2} + Z_L \), find

a) \( Z_{eq} = \frac{Z_1Z_2}{Z_1+Z_2} \) in polar form.

b) \( I \) the complex current in polar form

c) \( i(t) \) the total current as a function of time
6. A spring-mass system with $m = 0.5$ (slugs), $k = 32$ (lb/ft), and no damping is shown in the diagram. The system is given an initial displacement of $x_0 = 0.4$ (ft) and initial velocity of $v_0 = 2.4$ (ft/s). Find (a) $x(t)$ as a single, phase shifted sine function, (b) the time when the mass first reaches its largest displacement, c) $T$ the period of the oscillation, and d) $v(t) = \frac{dx}{dt}$.

7. The motion of a mass spring damper system is given by the equation $
 x(t) = e^{-2t} [3 \sin(10t) + 4 \cos(10t)] \text{ (inches)}.
$

a) Express $x(t)$ using a single, phase-shifted sine function.

b) Find the velocity function $v(t) = \frac{dx}{dt}$. Express your result using a single, phase shifted sine function.

8. A car has the acceleration profile shown. Its initial position is $s(0) = 0$ and its initial velocity is $v(0) = 5$ (m/s).

a) Find the velocity function $v(t)$ given that $v(t) = \int a(t) dt$.

b) Find the displacement function $s(t)$ given that $s(t) = \int v(t) dt$.

c) Find the velocity of the car at $t = 20$ (s).

d) Find the total distance traveled by the car for $0 \leq t \leq 20$ (s).

e) Sketch the velocity and position functions given that

$$\Delta v_{t_1}^{t_2} = \int_{t_1}^{t_2} a(t) dt = \text{Area under the acceleration profile from } t_1 \text{ to } t_2$$

$$\Delta s_{t_1}^{t_2} = \int_{t_1}^{t_2} v(t) dt = \text{Area under the velocity profile from } t_1 \text{ to } t_2$$

9. A voltage $v(t) = 75 e^{-3t}$ (volts) is applied to an inductor with inductance $L = 250$ (mh). Find the current $i(t)$, given that $i(t) = \frac{1}{L} \int v(t) dt$. Assume $i(0) = 0$. 
