ME 2560 Statics
Moment of a Force about an Axis

Two Dimensional Systems

- The moment of a force about a point $O$ may be calculated using a cross product.
  \[ M_O = r \times \vec{F} \]

- Here, $M_O$ is perpendicular to the plane formed by the vectors $\vec{r}$ and $\vec{F}$, and it has magnitude $|M_O| = |\vec{F}| d = |\vec{F}| r \sin(\theta)$.

- In the two dimensional system shown, $M_O$ represents the moment of the force about an axis perpendicular to the page (in the $\vec{k}$ direction) and passing through $O$.

![Diagram of moment in two dimensions](image)

Three Dimensional Systems

- To find the moment of a force about an axis in three dimensional analysis, we first calculate the moment about any point on that axis, say $O$, then we project that moment onto the axis using the dot product.
  \[ M_{\text{\textbar{g}-axis}} = M_O \cdot \vec{n} = (r \times \vec{F}) \cdot \vec{n} \]

- Here, $M_{\text{\textbar{g}-axis}}$ is the scalar moment of $\vec{F}$ about the axis passing through $O$ and parallel to the unit vector $\vec{n}$. In vector form, we write $M_{\text{\textbar{g}-axis}} = (M_O \cdot \vec{n}) \vec{n}$.

- $M_{\text{\textbar{g}-axis}}$ can be positive or negative depending on the angle between $\vec{n}$ and $M_O$. If it is positive, point your right thumb in the direction of $\vec{n}$ and your right fingers will show the circulation of $\vec{F}$ about the axis. If it is negative, point your right thumb opposite the direction of $\vec{n}$ and your right fingers will show the circulation of $\vec{F}$ about the axis.

- As before, $\vec{r}$ is a position vector from $O$ to any point on the line of action of $\vec{F}$.
Example:

Given: \( F = -100 \hat{i} + 50 \hat{j} + 200 \hat{k} \) (lb); \( A: (3,4,5) \) (ft)

Find: \( a) \ M_x, \ M_y, \ \text{and} \ M_z \) the moments of \( F \) about the \( X, Y, \) and \( Z \) axes, \( b) \ M_{n-axis} \) the scalar moment of \( F \) about an axis in the \( X-Y \) plane that makes an angle of 30 (deg) with the \( X \)-axis, and \( c) \ M_{\hat{n}-axis} \)

Solution:

\( a) \ M_O = r_{AO} \times F \)

\[
\begin{vmatrix}
i & j & k \\
3 & 4 & 5 \\
-100 & 50 & 200 \\
\end{vmatrix} = (800 - 250) \hat{i} - (600 + 500) \hat{j} + (150 + 400) \hat{k}
\]

\[
= 550 \hat{i} - 1100 \hat{j} + 550 \hat{k} \text{ (ft-lb)}
\]

\[
M_x = M_O \cdot \hat{i} = 550 \text{ (ft-lb)}, \quad M_y = M_O \cdot \hat{j} = -1100 \text{ (ft-lb)}, \quad M_z = M_O \cdot \hat{k} = 550 \text{ (ft-lb)}
\]

\( b) \ n = \cos(30) \hat{i} + \sin(30) \hat{j} \)

\[
M_{n-axis} = M_O \cdot n = (550 \cos(30)) + (-1100 \cdot \sin(30)) + (550 \cdot 0) = -73.686 \approx -73.7 \text{ (ft-lb)}
\]

\( c) \ M_{\hat{n}-axis} = (M_O \cdot n) n = -73.686 n = -63.8 \hat{i} - 36.8 \hat{j} \text{ (ft-lb)}
\]

Note on Calculation of the Scalar Moment: \( M_{\hat{n}-axis} \)

- Calculation of the scalar moment can also be done in determinant form. Simply replace the first row of the determinant by the components of \( n \) and expand as usual.

\[
M_{\hat{n}-axis} = \begin{vmatrix}
n_x & n_y & n_z \\
r_x & r_y & r_z \\
F_x & F_y & F_z \\
\end{vmatrix} = n_x (r_y F_z - r_z F_y) - n_y (r_x F_z - r_z F_x) + n_z (r_x F_y - r_y F_x)
\]

- So, for the above example, we have

\[
M_{\hat{n}-axis} = \begin{vmatrix}
\cos(30) & \sin(30) & 0 \\
3 & 4 & 5 \\
-100 & 50 & 200 \\
\end{vmatrix} = \cos(30)(800 - 250) - \sin(30)(600 + 500)
\]

\[
\approx -73.7 \text{ (ft-lb)}
\]