ME 2580 Dynamics
Conservation of Momentum and Impact for Rigid Bodies (2D)

Conservation of Linear Momentum

If the net impulse on a rigid body is zero over the time interval \( t_1 \rightarrow t_2 \), then the linear momentum of the body is conserved (i.e. the mass center moves with constant velocity). If the net impulse on a system of rigid bodies is zero over the time interval \( t_1 \rightarrow t_2 \), then the linear momentum of the system of bodies is conserved.

For example, consider two colliding bodies. If the impulsive forces \( \vec{F} \) and \( -\vec{F} \) act over a short time interval \( t_1 \rightarrow t_2 \), then the principle of linear impulse and momentum can be applied to each body over that interval. Because the impulses of \( \vec{F} \) and \( -\vec{F} \) are equal and opposite, the linear momentum of the system of the two bodies is conserved. That is,

\[
(m_A \vec{v}_A)_1 + (m_B \vec{v}_B)_1 = (m_A \vec{v}_A)_2 + (m_B \vec{v}_B)_2
\]

Conservation of Angular Momentum

If the net angular impulse on a rigid body is zero over the time interval \( t_1 \rightarrow t_2 \), then the angular momentum of the body is conserved (i.e. the body rotates with constant angular velocity). If the net angular impulse on a system of rigid bodies is zero over the time interval \( t_1 \rightarrow t_2 \), then the angular momentum of the system of bodies is conserved. For example, consider again two colliding bodies. The net angular impulse of the two impulsive forces \( \vec{F} \) and \( -\vec{F} \) about the fixed point \( O \) is zero, so the angular momentum of the system about \( O \) is conserved during the impact. That is,

\[
(H_O)_{A1} + (H_O)_{B1} = (H_O)_{A2} + (H_O)_{B2}
\]
Coefficient of Restitution

Consider two colliding bodies $A$ and $B$. At the contact point $C$ the directions $n$ and $t$ are normal and tangent to the colliding surfaces. If the friction forces resulting from the impact are negligible, it can be shown that the relative velocities of the points of contact on the two bodies in the $n$-direction are related as follows:

$$e = \frac{(v_{CB})_{n2} - (v_{CA})_{n2}}{(v_{CA})_{n1} - (v_{CB})_{n1}}$$

Here, $(v_{CA})_{n1}$ and $(v_{CB})_{n1}$ represent the velocities of the contact points on bodies $A$ and $B$ in the $n$-direction before impact (time, $t_1$), and $(v_{CA})_{n2}$ and $(v_{CB})_{n2}$ represent the velocities of the contact points on bodies $A$ and $B$ in the $n$-direction after impact (time, $t_2$).