General Concepts:

**Position, Velocity, and Acceleration**

If a particle does not move in a straight line, then its motion is said to be *curvilinear*. Given \( \mathbf{r}(t) \) the position vector of a particle \( P \), the velocity and acceleration of \( P \) are defined to be

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \text{and} \quad \mathbf{a} = \frac{d\mathbf{v}}{dt}.
\]

The velocity \( \mathbf{v} \) is *tangent* to the path of \( P \) at all times. The acceleration \( \mathbf{a} \) is generally not tangent to the path.

**Rectangular Components**

If we use rectangular components, then the position, velocity, and acceleration vectors may be written as

\[
\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}
\]

\[
\mathbf{v}(t) = \dot{x}(t)\mathbf{i} + \dot{y}(t)\mathbf{j} + \dot{z}(t)\mathbf{k}
\]

\[
\mathbf{a}(t) = \ddot{x}(t)\mathbf{i} + \ddot{y}(t)\mathbf{j} + \ddot{z}(t)\mathbf{k}
\]

*Note* that the methods for straight-line (rectilinear) motion can be applied in each direction.
Example: The Projectile Problem

If we *neglect air resistance*, the motion of a projectile can be analyzed using the equations for constant acceleration. The *horizontal motion* (X-direction) occurs at a *constant velocity*, and the *vertical motion* (Y-direction) occurs at a *constant acceleration*. The equations that apply in the X and Y directions are

X-direction: (constant velocity, $v_{x0}$)

$$x(t) = x_0 + v_{x0} \cdot t$$

Y-direction: (constant acceleration, $-g$)

$$v_y(t) = v_{y0} - gt$$  \quad $$y(t) = y_0 + v_{y0} \cdot t - \frac{1}{2} gt^2$$  \quad $$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

Derivative of a Rotating Unit Vector

Given the unit vector

$$\mathbf{e} = \cos(\theta) \mathbf{i} + \sin(\theta) \mathbf{j}$$

we can *differentiate* with respect to time to get

$$\dot{\mathbf{e}} = -\dot{\theta} \sin(\theta) \mathbf{i} + \dot{\theta} \cos(\theta) \mathbf{j} = \dot{\theta}(-\sin(\theta) \mathbf{i} + \cos(\theta) \mathbf{j}) = \dot{\theta} \mathbf{e}_\perp$$

But, we also note that $\mathbf{e}_\perp = k \times \mathbf{e}$, so that we can write

$$\dot{\mathbf{e}} = \dot{\theta}(k \times \mathbf{e}) = (\dot{\theta}k) \times \mathbf{e} = \omega \times \mathbf{e}$$

Here, $\mathbf{\omega}$ is the *angular velocity* of the unit vector set $(\mathbf{e}, \mathbf{e}_\perp)$. 