Definition of Mass Moment of Inertia

The figure depicts a rigid body in two dimensions. The moment of inertia of the body about the Z-axis is defined as

$$I_Z = \int r^2 \, dm = \int \left( x^2 + y^2 \right) \, dm.$$  

The moment of inertia $I_Z$ measures the distribution of mass about the Z-axis. The larger the inertia, the further the mass is spread away from the axis. The smaller the inertia, the closer the mass is located to the axis. The units of $I_Z$ are slug-ft$^2$ or kg-m$^2$.

Parallel Axes Theorem

The moment of inertia of the body about the Z-axis can be related to the moment of inertia about an axis parallel to Z and passing through $G$ as follows:

$$I_Z = \int \left( x^2 + y^2 \right) \, dm = \int \left( (x_G + x')^2 + (y_G + y')^2 \right) \, dm$$

$$= \int \left( x_G^2 + y_G^2 \right) \, dm + \int 2x_G x' \, dm + \int 2y_G y' \, dm + \int \left( x'^2 + y'^2 \right) \, dm$$

$$= \left( x_G^2 + y_G^2 \right) \int \, dm + 2x_G \int x' \, dm + 2y_G \int y' \, dm + (I_Z)_G$$

$$= M d^2 + (I_Z)_G$$

or

$$I_Z = (I_Z)_G + M d^2$$  

or  

$$I_O = I_G + M d^2$$  

(Parallel Axes Theorem)
Mass Moments of Inertia for Composite Shapes

The mass moment of inertia of a composite shape is the sum of the inertias of the individual component shapes. If $G$ is the mass center of the composite shape, then

$$I_G = \sum_i (I_G)_i$$

where $(I_G)_i$ represents the moment of inertia of the $i^{th}$ component shape about the composite mass center $G$ and may be calculated as follows

$$(I_G)_i = (I_{G_i}) + m_i d_i^2.$$

Here, $I_{G_i}$ represents the moment of inertia of the $i^{th}$ component shape about its own mass center $G_i$ and can usually be found in the tables for common geometric shapes.