ME 2580 Dynamics
Relative Motion of Two Particles

The figure shows the paths of motion of two particles A and B. The vectors $\mathbf{r}_A$ and $\mathbf{r}_B$ represent the position vectors of A and B relative to a fixed point $O$, and the vector $\mathbf{r}_{A/B}$ represents the position vector of A relative to B. (Note that $\mathbf{r}_{A/B}$ starts at B and ends at A.)

Sometimes it is convenient to express the motion of a point relative to another moving point. For example, we can express the motion of point A relative to point B as follows: First, note that

$$ \mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B} \quad \text{or} \quad \mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B. $$

Then, we can differentiate this expression to define what we mean by the terms “relative velocity” and “relative acceleration.”

$$ \mathbf{v}_{A/B} = \frac{d}{dt} \mathbf{r}_{A/B} = \mathbf{v}_A - \mathbf{v}_B \quad \text{“relative velocity”} $$

$$ \mathbf{a}_{A/B} = \frac{d^2}{dt^2} \mathbf{r}_{A/B} = \mathbf{a}_A - \mathbf{a}_B \quad \text{“relative acceleration”} $$

In words, the motion (velocity or acceleration) of A relative to B represents the motion of A as seen by an observer translating with B. This concept is used extensively in the analysis of rigid body kinematics.