General Concepts

Position, Velocity, and Acceleration

If a particle does not move in a straight line, then its motion is said to be curvilinear. Given \( \mathbf{r}(t) \) the position vector of a particle \( P \), the velocity and acceleration of \( P \) are defined to be

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \text{and} \quad \mathbf{a} = \frac{d\mathbf{v}}{dt}.
\]

The velocity \( \mathbf{v} \) is tangent to the path of \( P \) at all times. The acceleration \( \mathbf{a} \) is not usually tangent to the path.

Cartesian Coordinates

If we use Cartesian coordinates, then the position, velocity, and acceleration vectors may be written as

\[
\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}
\]

\[
\mathbf{v}(t) = \dot{x}(t)\mathbf{i} + \dot{y}(t)\mathbf{j} + \dot{z}(t)\mathbf{k}
\]

\[
\mathbf{a}(t) = \ddot{x}(t)\mathbf{i} + \ddot{y}(t)\mathbf{j} + \ddot{z}(t)\mathbf{k}
\]

Note that the methods for straight-line motion can be applied in each direction.
**Example: The Projectile Problem**

If we neglect air resistance, the motion of a projectile can be analyzed using the equations for constant acceleration. The horizontal motion (X-direction) occurs at a constant velocity, and the vertical motion (Y-direction) occurs at a constant acceleration. The equations that apply in the X and Y directions are

**X-direction**: (constant velocity, \( v_{x0} \))

\[ x(t) = x_0 + v_{x0} t \]

**Y-direction**: (constant acceleration, \(-g\))

\[ v_y(t) = v_{y0} - gt \]
\[ y(t) = y_0 + v_{y0} t - \frac{1}{2} gt^2 \]
\[ v_y^2 = v_{y0}^2 - 2g(y - y_0) \]

**Derivative of a Rotating Unit Vector**

Given the unit vector

\[ \vec{e} = \cos(\theta)\hat{i} + \sin(\theta)\hat{j} , \]

we can differentiate with respect to time to get

\[ \dot{\vec{e}} = -\dot{\theta}\sin(\theta)\hat{i} + \dot{\theta}\cos(\theta)\hat{j} = \dot{\theta}\left(-\sin(\theta)\hat{i} + \cos(\theta)\hat{j}\right) = \dot{\theta}\vec{e}_\perp \]

But, we also note that \( \vec{e}_\perp = k \times \vec{e} \), so that we can write

\[ \dot{\vec{e}} = \dot{\theta}(k \times \vec{e}) = (\dot{\theta}k) \times \vec{e} = \vec{\omega} \times \vec{e} \]

Here, \( \vec{\omega} \) is the angular velocity of the unit vector set \((\vec{e}, \vec{e}_\perp)\).