Solve the following problems using Lagrange’s equations. Use the generalized coordinates suggested for each problem.

1. Find the equation of motion of the single degree-of-freedom, spring-mass-damper system shown. It is driven by the force $F(t) = A \sin(\omega t)$ and the spring is unstretched when $x = 0$. Use $x$ as the generalized coordinate. Neglect friction.

2. Find the equation of motion of the single degree-of-freedom inverted pendulum shown. The bar has mass $m = 80$ (kg) and length $\ell = 3$ (m). The bar is acted on by gravity and the horizontal spring of stiffness $k = 2$ (kN/m). The spring is unstretched when $\theta = 90$ (deg). Use $\theta$ as the generalized coordinate.

3. Find the equation of motion of the single degree-of-freedom system shown. The system consists of bar $AB$ of mass $m$ and length $\ell$ and a piston $P$ of mass $m_p$. The system is driven by the force $F(t) = F_0 + A \sin(\omega t)$ and gravity. A spring and damper are attached to the massless slider at $B$. The spring is unstretched when $x = 0$. Use $\theta$ as the generalized coordinate. Neglect friction.

4. Find the equations of motion of the two degree-of-freedom spring-mass-damper system shown. It is driven by the force $F(t) = A \sin(\omega t)$ acting on mass $m_1$. The springs are unstretched when $x_1 = x_2 = 0$. Use $x_1$ and $x_2$ as the generalized coordinates. Neglect friction.

5. Find the equations of motion of the two degree-of-freedom system shown. The system consists of a mass $m_1$ that moves along a fixed horizontal bar and bar $AB$ that is pinned to mass $m_1$ at $A$. Bar $AB$ has mass $m_2$ and length $\ell$. Mass $m_1$ is attached to the fixed support by a damper and a spring that is unstretched when $x = 0$. The system is driven by the force $F(t) = A \sin(\omega t)$ applied to $m_1$ and gravity. Use $x$ and $\theta$ as the generalized coordinates. Neglect friction.