One major concern of control system designers is: "Can the physical system generate the control effort required to give the desired performance?"

To check this, the designer must understand not only the total closed-loop system response, but also the response of individual components of the system and how the components interact.

Consider the aircraft attitude control system shown in the block diagram below. To ensure proper performance of the plant (the aircraft), the control signal $U(s)$ (output of the compensator) must be in the appropriate range for proper plant operation.

If it is too large, the “real” plant may not provide the proper response. In this case, if the wing flaps are commanded to too large an angle, then the flap will "stall" and not produce the forces necessary to control the aircraft.

Example: Control Effort of a PID Compensator

Previously, we designed a PID compensator and pre-filter for this system of the form

$$G_c(s) = \frac{K(s^2 + as + b)}{s} \quad \text{and} \quad G_{pf}(s) = \frac{(1143)^3}{1639(s^2 + as + b)}$$

Here, $K = 0.36423$, $a = 1713.73$, and $b = 9.1107 \times 10^5$

To find the control signal $U(s)$, we first find the transfer function $U(s)/\theta_d(s)$ for systems with and without a pre-filter to be

$$\begin{align*}
\frac{U(s)}{\theta_d(s)} &= \frac{(3.31845 \times 10^5) s (s + 361.2)}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + 1.49327 \times 10^9} \\
\text{With Pre-Filter} & \quad (1)
\end{align*}$$
\[
\frac{U(s)}{\theta_d(s)} = \frac{K s (s + 361.2) (s^2 + as + b)}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + 1.49327 \times 10^9}
\]

Without Pre-Filter \( (2) \)

- The transfer function for the system with a pre-filter is proper, while the transfer function for the system without a pre-filter is not proper. We cannot generate a step response using MATLAB for systems with improper transfer functions.

Control Signal with a Pre-Filter

- Assuming a step input for the desired attitude angle \( \theta_d(s) \), the control signal for the system with a pre-filter can be found in various ways. The figure to the right shows the signal found using the transfer function of Eq. (1).

- The system designer must now decide if that signal is within the range of acceptable inputs for the aircraft.

- The signal can also be found using the following Simulink model.
The results for the control input and attitude angle are shown below.

Control Signal without a Pre-Filter

- As noted above, if we do not use a pre-filter, the transfer function from the desired attitude angle to the control signal is not a proper transfer function. In this case, it is convenient to use Simulink to estimate the control signal.
- Simulink model without pre-filter:
The results for the control input and attitude angle are shown below. Note there is a discontinuity in the control signal at $t = 0$ due to the derivative control term, so the first few points are omitted from the plot.

**Summary**

- The maximum value of the control signal for the system without a pre-filter is almost twice as large as the system with a pre-filter.
- By “softening” the input to the control loop, the pre-filter has reduced the load on the actuation system (flaps on the aircraft).
- Another way of looking at pre-filters is that they can be used to generate a command input (or command trajectory) that is easier for the system to follow. As a direct result of this feature, it reduces the load on the actuation system. If properly designed, pre-filters can be used to avoid saturation of the actuation system.