Phase Lag Position Control of a Spring-Mass-Damper with RL Diagrams

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This file produces root locus diagram and step responses for Phase Lag position control of a spring-mass-damper system.

Clear Variables and start pause feature

```
clear m c kspring z p alpha K poles numP denP sysCOL sysCCL sysUCL
clear titlestringRL titlestring performanceCharacteristics
disp('Phase Lag Position Control of a Spring-Mass-Damper with RL Diagrams');

%  turn on pause feature
pause on;
```

Define parameters of the SMD

```
m  = 1.0;  %% mass in slugs
    c  = 2.0;  %% damping coefficient in lb-s/ft
```

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kspring = 2.0;  % spring stiffness in lb/ft

**First Try: Define the pole and zero locations for the compensator**

z and p are the zero and pole of the candidate phase-lag compensator. For the compensator to be phase-lag, p < z. If p > z, the compensator is phase-lead and a warning message is sent to the command window.

```
z = 0.5;  % zero for Phase Lag compensator
p = 0.1;  % pole for Phase Lag compensator (p<z)
if (p > z)
    disp('Warning: This is a Phase-Lead Compensator')
end
```

**Plot the root locus diagram for the compensated system**

This calculation uses the compensated open-loop transfer function GH(s). Observe the RL diagram to determine if poles can be selected in favorable target regions.

```
% define the transfer function of the compensated, open-loop system
alpha = p/z;
numP = alpha*[1,z]; denP = conv([1,p],[m,c,kspring]); sysCOL = tf(numP,denP);
% plot the root locus diagram of the compensated system in figure window #1
figure(1); clf;
rlocus(sysCOL); grid on;
titlestringRL = ['Root Locus Diagram for K. Phase Lag Compensator with Zero = num2str(z),', 'Pole = ',num2str(p)];
title(titlestringRL);
```
Find a suitable "K" for the closed loop system

Use your mouse to click on poles locations that you think are favorable. Calculate gain K and all associated poles and display them in the command window.

```matlab
% pause to allow user to zoom into areas of interest
pause;

% call rlocfind to allow user to pick the pole locations
[K,poles] = rlocfind(sysCOL);

% display the gain and associated poles in Command Window
disp('Selected gain K and the associated poles');
disp('=========================================');
disp('K = '); disp(K);
disp('Closed loop poles = '); disp(poles);

Select a point in the graphics window

selected_point =
-0.857751277683135 + 2.12655601659751i

Selected gain K and the associated poles
=========================================
\[ K = 18.6023899326382 \]

Closed loop poles =

\[-0.85380255005916 + 2.12636865565257i \]
\[-0.85380255005916 - 2.12636865565257i \]
\[-0.392394899881681 \]

Plot step response of compensated closed loop system with selected \( K \)

If this response does not meet specifications, then try different pole and zero values for the compensator. The idea is to manipulate the shape of the RL diagram to force the closed-loop poles into favorable target regions. This calculation uses the compensated closed-loop transfer function.

\[
\text{% define the transfer function for the compensated, closed-loop system with gain and negative unity feedback}
\text{sysCL} = \text{feedback}(K*\text{sysCL},1,-1);
\]

\[
\text{% plot the step response of the compensated, closed-loop system on the specified time interval in figure window #2}
\text{figure(2); clf; hold on;}
\]
Phase Lag Position Control of a Spring-Mass-Damper with RL Diagrams

\[ t = [0:0.01:20]; \]
\[ \text{step(sysCCL, } t); \text{ grid on;} \]
\[ \text{ylabel('Position Change (ft)');} \]
\[ \text{titlestring = ['Step Response of SMD with Phase Lag Control. } K = ' , ... \]
\[ \text{num2str(K), ' Zero = ',num2str(z), ', Pole = ',num2str(p)];} \]
\[ \text{title(titlestring);} \]

Plot step response of Uncompensated closed loop system with selected K

This calculation uses the uncompensated closed-loop transfer function. For comparison, the result is plotted on the same graph as the compensated system.

% define the transfer function of the uncompensated, closed-loop system
% with gain K and negative unity feedback
numPU = 1; denPU = [m,c,kspring]; sysCOLU = tf(numPU,denPU);
sysUCL = feedback(K*sysCOLU,1,-1);

% plot the step response of the uncompensated, closed-loop system
step(sysUCL, t);
legend('Phase Lag Compensated','Uncompensated')

% pause to allow user to annotate the plot
pause;
Calculate and display the performance characteristics (performanceCharacteristics)

% calculate performance characteristics; stored in structure: performanceCharacteristics
performanceCharacteristics = stepinfo(sysCCL);

% display results to Command Window
disp('Performance Data for the Compensated System');
disp('===========================================');
disp('Rise Time (sec) ='); disp(performanceCharacteristics.RiseTime);
disp('Settling Time (sec) ='); disp(performanceCharacteristics.SettlingTime);
disp('Percent Overshoot ='); disp(performanceCharacteristics.Overshoot);
disp('Peak Value ='); disp(performanceCharacteristics.Peak);
disp('Peak Time (sec) ='); disp(performanceCharacteristics.PeakTime);

Performance Data for the Compensated System
===========================================
Rise Time (sec) = 0.786693278799929
Settling Time (sec) = 6.55920993148586
Percent Overshoot = 8.46120061923741
Peak Value = 0.979322084999868
Peak Time (sec) =
Plot the Bode diagram of the closed loop system

```matlab
figure(3); clf;
bode(sysCCL); grid on;
titlestring = ['Bode Diagram of Phase-Lag Compensated, Closed Loop SMD. K = ', num2str(K), ', Zero = ',num2str(z), ', Pole = ',num2str(p)];
title(titlestring);
pause;
%
calculate and display the bandwidth (BW) and low frequency gain (DC_gain) of the compensated, closed-loop system
BW = bandwidth(sysCCL); DC_gain = 20*log10(dcgain(sysCCL));
disp('DC Gain and Bandwidth of the Compensated, Closed-loop System');
disp('============================================================');
disp('DC Gain (dB) = '); disp(DC_gain);
disp('Bandwidth (rad/s) = '); disp(BW);
```

DC Gain and Bandwidth of the Compensated, Closed-loop System
============================================================

DC Gain (dB) =
-0.886977181059902

Bandwidth (rad/s) =
2.89486085993467
Second Try: Define the pole and zero locations for the compensator

$z$ and $p$ are the zero and pole of the candidate phase-lag compensator. For the compensator to be phase-lag, $p < z$. If $p > z$, the compensator is phase-lead and a warning message is sent to the command window.

$$z = 0.25; \quad \text{\% zero for Phase Lag compensator}$$

$$p = 0.05; \quad \text{\% pole for Phase Lag compensator (p<z)}$$

if $(p > z)$
    disp('Warning: This is a Phase-Lead Compensator')
end

Plot the root locus diagram for the compensated system

This calculation uses the compensated open-loop transfer function $GH(s)$. Observe the RL diagram to determine if poles can be selected in favorable target regions.

% define the transfer function of the compensated, open-loop system
alpha = p/z;
numP = alpha*[1,z]; denP = conv([1,p],[m,c,kspring]); sysCOL = tf(numP,denP);
% plot the root locus diagram of the compensated system in figure window #4
figure(4); clf;
Find a suitable "K" for the closed loop system

Use your mouse to click on poles locations that you think are favorable. Calculate gain K and all associated poles and display them in the command window.

```matlab
% pause to allow user to zoom into areas of interest
pause;
% call rlocfind to allow user to select pole locations of interest
[K,poles] = rlocfind(sysCOL);
% display the selected gain and associated poles in Command Window
disp('Selected gain K and the associated poles');
disp('=========================================');
disp('K = '); disp(K);
disp('Closed loop poles = '); disp(poles);
```
Selected gain $K$ and the associated poles

\[ K = 18.409505967953 \]

Closed loop poles =

\[-0.93106952811599 + 2.13662994617071i \]
\[-0.93106952811599 - 2.13662994617071i \]
\[-0.187860943768017 \]

Plot step response of compensated closed loop system with selected $K$

If this response does not meet specifications, then try different pole and zero values for the compensator. The idea is to manipulate the shape of the RL diagram to force the closed-loop poles into favorable target regions. This calculation uses the compensated closed-loop transfer function.

% define the transfer function of the compensated, closed-loop system with gain
% and negative unity feedback
sysCCL = feedback(K*sysCOL,1,-1);

% plot the step response of the compensated, closed-loop system in figure window #5
figure(5); clf; hold on;
step(sysCCL, t); grid on;
ylabel('Position Change (ft)');
titlestring = ['Step Response of SMD with Phase Lag Control. K = ', ...
num2str(K), ', Zero = ',num2str(z), ', Pole = ',num2str(p)];
title(titlestring);

Plot step response of Uncompensated closed loop system with selected K

This calculation uses the uncompensated closed-loop transfer function. For comparison, the result is plotted on the same graph as the compensated system.

% define the transfer function of the uncompensated, closed-loop system with gain and negative unity feedback
numPU = 1; denPU = [m,c,kspring]; sysOLU = tf(numPU,denPU);
sysUCL = feedback(K*sysOLU,1,-1);

% plot the step response of the uncompensated, closed-loop system
step(sysUCL, t);
legend('Phase Lag Compensated','Uncompensated')
Calculate and display the performance characteristics (performanceCharacteristics)

% calculate performance characteristics; stored in structure: performanceCharacteristics
performanceCharacteristics = stepinfo(sysCCL);

% display results to Command Window
disp('Performance Data for the Compensated System');
disp('---------------------------------------------');
disp('Rise Time (sec) ='); disp(performanceCharacteristics.RiseTime);
disp('Settling Time (sec) ='); disp(performanceCharacteristics.SettlingTime);
disp('Percent Overshoot ='); disp(performanceCharacteristics.Overshoot);
disp('Peak Value ='); disp(performanceCharacteristics.Peak);
disp('Peak Time (sec) ='); disp(performanceCharacteristics.PeakTime);

Performance Data for the Compensated System
---------------------------------------------
Rise Time (sec) = 0.893440704557809
Settling Time (sec) = 13.7309158972112
Percent Overshoot = 0
Peak Value = 0.901586632877494
Peak Time (sec) = 33.7490306962988

Plot the Bode diagram of the closed loop system

```
figure(6); clf;
bode(sysCCL); grid on;
titlestring = ['Bode Diagram of Phase-Lag Compensated, Closed Loop SMD. K = ', num2str(K), ', Zero = ', num2str(z), ', Pole = ', num2str(p)];
title(titlestring);
```

```
% pause to allow user to annotate plot
pause;
```

```
% calculate and display the bandwidth (BW) and low frequency gain (DC_gain) of the compensated, closed-loop system
BW = bandwidth(sysCCL); DC_gain = 20*log10(dcgain(sysCCL));
disp('DC Gain and Bandwidth of the Compensated, Closed-loop System');
disp('============================================================');
disp('DC Gain (dB) ='); disp(DC_gain);
disp('Bandwidth (rad/s) ='); disp(BW);
```

DC Gain and Bandwidth of the Compensated, Closed-loop System
============================================================
DC Gain (dB) = -0.895807165167528
Bandwidth (rad/s) = 2.80741180531432
Third Try: Define the pole and zero locations for the compensator

z and p are the zero and pole of the candidate phase-lag compensator. For the compensator to be phase-lag, \( p < z \). If \( p > z \), the compensator is phase-lead and a warning message is sent to the command window.

```matlab
z = 0.25; % zero for Phase Lag compensator
p = 0.05; % pole for Phase Lag compensator (p<z)
if (p > z)
    disp('Warning: This is a Phase-Lead Compensator')
end
```

Plot the root locus diagram for the compensated system

This calculation uses the compensated open-loop transfer function \( GH(s) \). Observe the RL diagram to determine if poles can be selected in favorable target regions.

```matlab
% define the transfer function of the compensated, open-loop system
```
alpha  = p/z;
numP = alpha*[1,z]; denP = conv([1,p],[m,c,kspring]); sysCOL = tf(numP,denP);

% plot the root locus diagram of the compensated system in figure window #7
figure(7); clf;
rlocus(sysCOL); grid on;
titlestringRL = ['Root Locus Diagram for K. Phase Lag Compensator with Zero = 
num2str(z),', ' Pole = ',num2str(p)];
title(titlestringRL);

Find a suitable "K" for the closed loop system

Use your mouse to click on poles locations that you think are favorable. Calculate gain K and all associated poles and display them in the command window.

% pause to allow user to zoom into areas of interest
pause;

% call rlocfind to allow user to select pole locations of interest
[K,poles] = rlocfind(sysCOL);

% display the selected gain and associated poles in Command Window
disp('Selected gain K and the associated poles');
disp('========================================');
disp('K = '); disp(K);
disp('Closed loop poles = '); disp(poles);

Select a point in the graphics window

selected_point =
Selected gain $K$ and the associated poles

$K = 28.0997209172602$

Closed loop poles = 
-0.922504738165297 + 2.54768909657097i
-0.922504738165297 - 2.54768909657097i
-0.204990523669402

Plot step response of compensated closed loop system with selected $K$

If this response does not meet specifications, then try different pole and zero values for the compensator. The idea is to manipulate the shape of the RL diagram to force the closed-loop poles into favorable target regions. This calculation uses the compensated closed-loop transfer function.

```matlab
% define the transfer function of the compensated, closed-loop system with gain and negative unity feedback
sysCCL = feedback(K*sysCOL,1,-1);

% plot the step response of the compensated, closed-loop system in figure window #8
figure(8); clf; hold on;
step(sysCCL, t); grid on;
```
ylabel('Position Change (ft)');
titlestring = ['Step Response of SMD with Phase Lag Control. K = ', ...
num2str(K), ', Zero = ',num2str(z), ', Pole = ',num2str(p)];
title(titlestring);

Plot step response of Uncompensated closed loop system with selected K

This calculation uses the uncompensated closed-loop transfer function. For comparison, the result is plotted on the same graph as the compensated system.

% define the transfer function of the uncompensated, closed-loop system with gain and negative unity feedback
numPU = 1; denPU = [m,c,kspring]; sysOLU = tf(numPU,denPU);
sysUCL = feedback(K*sysOLU,1,-1);

% plot the step response of the uncompensated, closed-loop system
step(sysUCL, t);
legend('Phase Lag Compensated','Uncompensated')

% pause to allow user to annotate the plot
pause;
Calculate and display the performance characteristics (performanceCharacteristics)

Calculate and display the performance characteristics (performanceCharacteristics)

```matlab
% calculate performance characteristics; stored in structure: performanceCharacteristics
performanceCharacteristics = stepinfo(sysCCL);

% display results to Command Window
disp('Performance Data for the Compensated System');
disp('===========================================');
disp('Rise Time (sec) ='); disp(performanceCharacteristics.RiseTime);
disp('Settling Time (sec) ='); disp(performanceCharacteristics.SettlingTime);
disp('Percent Overshoot ='); disp(performanceCharacteristics.Overshoot);
disp('Peak Value ='); disp(performanceCharacteristics.Peak);
disp('Peak Time (sec) ='); disp(performanceCharacteristics.PeakTime);
```

Performance Data for the Compensated System
===========================================
Rise Time (sec) =
0.624022368850526

Settling Time (sec) =
10.9421248395185

Percent Overshoot =
11.4029447016764
Peak Value =
1.0400687052318

Peak Time (sec) =
1.25103848639583

Plot the Bode diagram of the closed loop system

```matlab
% plot the bode diagram of the compensated, closed-loop system in figure window
figure(9); clf;
bode(sysCCL); grid on;
titlestring = ['Bode Diagram of Phase-Lag Compensated, Closed Loop SMD. K = ', num2str(K), ', Zero = ', num2str(z), ', Pole = ', num2str(p)];
title(titlestring);

pause;

% calculate and display the bandwidth (BW) and low frequency gain (DC_gain) of the compensated, closed-loop system
BW = bandwidth(sysCCL); DC_gain = 20*log10(dcgain(sysCCL));
disp('DC Gain and Bandwidth of the Compensated, Closed-loop System');
disp('============================================================');
disp('DC Gain (dB) ='); disp(DC_gain);
disp('Bandwidth (rad/s) ='); disp(BW);

pause off;
```

DC Gain and Bandwidth of the Compensated, Closed-loop System
============================================================
DC Gain (dB) =
-0.59720924580765

Bandwidth (rad/s) =
3.56832797514371
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