**Phase-Lead Compensator** (PD type compensator)

- Transfer function of a *phase-lead* compensator: \( G_c(s) = \alpha \left( \frac{s + z}{s + p} \right) \), where \(|z| < |p|\).
- The multiplier \( \alpha = |p|/|z| \).
- As a result, the compensator *amplifies* at frequencies near and above the location of the zero, and *increases* the phase of the system near the pole and zero locations, as well.
- A Bode diagram of the phase-lead compensator \( G_c(s) = 10 \left( \frac{s + 1}{s + 10} \right) \) is shown below.

Here, \( \alpha = 10 \), the *logarithmic mean frequency* is \( \omega_m = \sqrt{|pz|} = 3.16 \) (rad/s), the overall magnitude increase is \( 20 \log(\alpha) = 20 \) (dB), and the phase shift at the mean frequency is \( \phi_m = \sin^{-1} \left( \frac{\alpha - 1}{\alpha + 1} \right) = +54.9 \) (deg).
Phase-Lead Compensator Design Using Bode Diagrams

- **Find loop gain** $K$ required to satisfy the steady-state error requirement (if given).

- Evaluate the **phase margin (PM)** of the uncompensated system with the loop gain $K$ to determine if proportional control is sufficient.

- **Find** $\phi_a$ the necessary additional phase required to give the desired phase margin.

- **Find** $\alpha$ for the compensator using the equation $\alpha = \frac{1 + \sin(\phi_a)}{1 - \sin(\phi_a)}$.

- Examine the **Bode plot** of the uncompensated system (with the loop gain $K$) to find the frequency where $M = -10\log(\alpha)$. Define this frequency to be $\omega_m$ the logarithmic mean frequency of the compensator. This will be the new zero dB crossover point.

- **Find** the pole and zero locations $p = \omega_m \sqrt{\alpha}$ and $z = p / \alpha$, and define the compensator to be $G_c(s) = K \alpha \left( \frac{s + z}{s + p} \right)$.

- **Check** the phase margin of the compensated system to see if the desired value has been attained. If not, then decide on the additional phase required, and repeat the steps above starting with the calculation of $\alpha$.

- **Simulate** the time-domain performance.

Phase-Lead Compensator Design Using Root Locus Diagrams

- **Target Regions**: Set damping ratio $\zeta$ and natural frequency $\omega_n$ values for the complex poles (assuming they are all dominant and have no influence from transfer function zeros) to target a desirable percent overshoot and settling time.

- **Examine** the uncompensated root locus diagram to see if the pole locations determined above can be met with only proportional control.

- **Add** a zero and pole to $GH(s)$ modifying the root locus diagram to move branches into the target region.
  - Changes the shape of the root locus diagram in predictable ways.
    - Can change the locations of real poles of the closed loop system.
    - Separation between the pole and zero will move all asymptotes to the left.
    - The location of any closed-loop zeros may cause overshoot problems.

- **Simulate** the time-domain performance.