ME 2580 Dynamics  
Rectilinear (Straight Line) Motion

General Concepts:

Position, Velocity, and Acceleration

A particle \( P \) has \textit{rectilinear} motion when it moves in a straight line. As shown in the figure, define the direction of motion as the \( X \)-axis along which we define a \textit{unit vector}, \( \vec{e} \).

The \textit{position vector} of \( P \) may then be written as \( \vec{r} = s \vec{e} \), where \( s \) is the distance from some fixed point on the axis (in this case, \( O \)) to \( P \). The \textit{velocity} of \( P \) is defined as the derivative of the position vector. Using the \textit{product rule}, we can write

\[
\frac{d}{dt} \vec{r} = \frac{d}{dt} (s \vec{e}) = \left( \frac{ds}{dt} \right) \vec{e} + \left( s \frac{d}{dt} \vec{e} \right) = \frac{ds}{dt} \vec{e} = \dot{s} \vec{e} = v \vec{e}
\]

Similarly, the \textit{acceleration} of \( P \) is defined as the derivative of the velocity. So, the acceleration may be written as

\[
\frac{d^2}{dt^2} \vec{r} = \ddot{s} \vec{e} = \ddot{v} \vec{e} = a \vec{e}
\]

Average Velocity, Average Speed, and Average Acceleration

If \( \Delta s \) and \( s_T \) represent the “displacement” and “total distance” traveled by \( P \) over the interval of time \( \Delta t \), then the average velocity and average speed of \( P \) over the interval \( \Delta t \) are defined to be

Average Velocity: \( v_{avg} = \frac{\Delta r}{\Delta t} \),  
Average Speed: \( v_{avg} = \frac{s_T}{\Delta t} \).

The average acceleration over the time interval \( \Delta t \) is defined to be: \( a_{avg} = \frac{\Delta v}{\Delta t} \).
Problem Solving:

1. Given: \( s = s(t) \)  
   Calculate: \( v(t) = \frac{ds}{dt} \) and \( a(t) = \frac{dv}{dt} \)

2. Given: \( a = a(t) \)  
   Calculate: \( \frac{dv}{dt} = a(t) \)  
   \[
   \begin{cases}
   v^0 \quad \text{and} \quad t_0 \\
   \int v \, dv = v(t) - v(0) = \int a(t) \, dt
   \end{cases}
   \]

3. Given: \( a = a(s) \)  
   Calculate: \( \frac{dv}{ds} = a(s) \)  
   \[
   \begin{cases}
   v^0 \quad \text{and} \quad s_0 \\
   v(s) \quad \text{and} \quad t_0 \\
   \int v \, dv = \frac{1}{2} (v^2 - v_0^2) = \int a(s) \, ds \\
   \int \frac{ds}{v(s)} = \int dt = t - t_0
   \end{cases}
   \]

4. Given: \( a = a(v) \)  
   Calculate: \( v \frac{dv}{ds} = a(v) \)  
   \[
   \begin{cases}
   v^0 \quad \text{and} \quad s_0 \\
   v(a(v)) \quad \text{and} \quad t_0 \\
   \int v \, dv = \int a(v) \, dt = t - t_0
   \end{cases}
   \]

5. Given: \( a = a(v) \)  
   Calculate: \( \frac{dv}{dt} = a(v) \)  
   \[
   \begin{cases}
   v^0 \quad \text{and} \quad t_0 \\
   s \quad \text{and} \quad t_0 \\
   \int ds = s(t) - s(0) = \int v(t) \, dt
   \end{cases}
   \]

6. Given: \( a = a_0 = \text{constant} \)  
   Calculate: \( \frac{dv}{dt} = a_0 \)  
   \[
   \begin{cases}
   v(t) = v_0 + a_0 (t - t_0) \\
   s(t) = s_0 + v_0 (t - t_0) + \frac{1}{2} a_0 (t - t_0)^2
   \end{cases}
   \]
   Calculate: \( v \frac{dv}{ds} = a_0 \)  
   \[
   v^2(t) = v_0^2 + 2 a_0 (s - s_0)
   \]