ME 5550 Intermediate Dynamics
Homework #3

1) The figure shows a three-dimensional slider crank mechanism. The \((x, y, z)\) axes shown represent a fixed reference frame \(R\). At the instant shown, disk \(A\) has angular velocity \(\omega_A = 10\hat{\mathbf{i}} \) (rad/sec).

   a) If \(BC\) is attached to both the disk and the collar with ball-and-socket joints, find \(\omega_{BC}\) and \(\nu_B\).

   In this case, assume that \(\omega_{BC}\) is perpendicular to \(BC\). (Hibbeler, 1995)

   b) If the ball-and-socket joint at the collar is replaced with the two-axis joint shown at the right, find \(\omega_{BC}\) and \(\nu_B\).

   In this case, the rod \(BC\) rotates relative to collar \(C\) about the axis defined by the unit vector \(\hat{n}\) which is perpendicular to the plane \(BCD\), and the collar translates and rotates about the \(\hat{j}\) direction. In this case, \(\omega_{BC}\) is perpendicular to the vector \(\mathbf{u} = \hat{j} \times \hat{n}\). (Hibbeler, 1995)
2) The system shown consists of a vertical column, a horizontal axle, and a wheel of radius $r$. The horizontal arm rotates at a constant rate $\Omega$, and the wheel ($W$) rolls without slipping in a circular arc. Find $\omega_W$ and $\alpha_W$ the angular velocity and angular acceleration of the wheel relative to a fixed frame, and find $v_A$ and $a_A$ the velocity and acceleration of point $A$. (Meriam and Kraige, 1992)

3) In the system shown, beveled gear $A$ rolls on beveled gear $B$. As it rolls on $B$ it spins about the axle $AD$ which is pinned to the vertical shaft $DE$. If $DE$ rotates at a constant angular velocity $\omega_1$ (rad/sec), find $\omega_A^{AD}$ the angular velocity of gear $A$ relative to the axle $AD$, $\omega_A^R$ the angular velocity of gear $A$, $\alpha_A^R$ the angular acceleration of gear $A$, and $a_C$ the acceleration of tooth $C$ of gear $A$. (Beer & Johnston)

4) The diagram below depicts a yoke-and-spider universal joint. The joint has three members – the input shaft $A$, the output shaft $B$, and the spider $S$. Shaft $A$ rotates about the $X$-axis, and shaft $B$ rotates about the $e_1$ direction which is in the $XZ$ plane and makes an angle of $\phi$ with the $X$-axis. At the instant shown, spider $S$ rotates relative to shaft $A$ about the direction of spider segment $EF$ (represented by unit vector $n_{EF} = N_2$), and it rotates relative to shaft $B$ about the spider segment $CD$ (represented by unit vector $n_{CD} = e_1 \times n_{EF} = e_1 \times N_2$). At the instant shown, find $\omega_B/\omega_A$ the ratio of the speed of shaft $B$ to the speed of shaft $A$. Hint: Apply the summation rule for angular velocities through the joint using the known directions for the angular velocities and relative angular velocities.
5) Referring again to the diagram of the yoke-and-spider **universal joint** of problem (4), let input shaft $A$ rotate through an angle $\theta$. The spider segment $EF$ is now at an angle $\theta$ to the $Y$-axis, so $n_{EF} = C_\theta \, N_2 + S_\theta \, N_3$. As before, shaft $B$ rotates about the $\zeta_1$ direction, and $n_{CD} = \epsilon_1 \times n_{EF}$. Find $\omega_B / \omega_A$ the ratio of the speed of shaft $B$ to the speed of shaft $A$ as a function of the input shaft angle $\theta$. Hint: Apply the **summation rule** for angular velocities **through the joint** using the known directions for the angular velocities and relative angular velocities.