ME 5550 Intermediate Dynamics
Equations of Motion of Example System II

In previous notes for Example System II, we found $^{R}\omega_{B}$ the angular velocity of the bar and $[I_{G}]_{c}$ that the inertia matrix (associated with $I_G$) resolved in bar-fixed directions $B: (e_1, e_2, e_3)$ to be

$$^{R}\omega_{B} = (-\Omega S_{\theta})e_1 + \omega n_2 + (\Omega C_{\theta})e_3$$

and

$$[I_{G}] = \frac{m\ell^2}{12} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The angular momentum of $B$ was also found to be

$$H_{G} = I_{G} \cdot ^{R}\omega_{B} = \frac{m\ell^2}{12} \left[ \omega n_2 + \Omega C_{\theta}e_3 \right]$$

The EOM (equations of motion) of $B$ can be found using the Newton/Euler equations along with the free-body diagrams shown at the right.

Given that $\Omega = \text{constant}$, the terms on the right side of the moment equation may be calculated as follows

$$^{R}\omega_{B} \times H_{G} = \frac{m\ell^2}{12} \begin{vmatrix} e_1 & n_2 & e_3 \\ -\Omega S_{\theta} & \omega & \Omega C_{\theta} \\ 0 & \omega & \Omega C_{\theta} \end{vmatrix} = \frac{m\ell^2}{12} \left( \Omega^2 S_{\theta}C_{\theta}n_2 - \omega\Omega S_{\theta}e_3 \right)$$

$$I_{G} \cdot ^{R}\alpha_{B} = \frac{m\ell^2}{12} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\omega\Omega C_{\theta} \\ \omega \end{bmatrix} = \frac{m\ell^2}{12} \left( \omega n_2 - \omega\Omega S_{\theta}e_3 \right)$$
Inverse Dynamics (assuming $\Omega$ and $\omega$ are constant)

\[
\begin{align*}
F_1 &= F_3 = 0 \\
F_2 &= -md\Omega^2 \\
T_1 &= 0 \\
T_2 &= \frac{1}{12} ml^2 \Omega^2 S_\theta C_\theta \\
T_3 &= -\frac{1}{6} ml^2 \omega \Omega S_\theta
\end{align*}
\]

Forward Dynamics of Bar $B$ (assuming $\Omega = \text{constant}$, $\omega = \dot{\theta}$, and $\dot{\omega} = \ddot{\theta}$)

\[
\begin{align*}
F_1 &= F_3 = 0 \\
F_2 &= -md\Omega^2 \\
T_1 &= 0 \\
\ddot{\theta} + \Omega^2 S_\theta C_\theta &= 12T_2/ml^2 \\
T_3 &= -\frac{1}{6} ml^2 \omega \Omega S_\theta
\end{align*}
\]

Note that the fourth equation is a \textit{differential equation} that describes the changes in the angle $\theta$.

Equilibrium Positions for the Bar

If $T_2(t) \equiv 0$, the bar exhibits \textit{equilibrium positions}. These positions may be calculated by setting all \textit{time-varying} parts of the differential equation to \textit{zero}. That is,

\[
\Omega^2 S_\theta C_\theta = 0
\]

This equation is satisfied when $\theta = 0, \pi/2$. In general, these equilibrium positions may be \textit{stable} or \textit{unstable}.