ME 6590 Multibody Dynamics
Conversion of Direction Cosines to 1-2-3 Body-Fixed Angle Sequence

Given the coordinate transformation matrix \([C]\), the orientation angles for a 1-2-3 body-fixed angle sequence may be computed as follows. First, recall that \([C]\) may be written as

\[
[C] = \begin{bmatrix}
    C_{23} S_{33} & C_{13} C_{3} & C_{13} S_{3} \\
    C_{33} C_{2} & C_{13} C_{3} & C_{13} S_{3} \\
    -S_{3} S_{33} & -S_{2} S_{33} & -S_{2} C_{3}
\end{bmatrix}
\]

The three orientation angles may then be calculated by observation

\[
\theta_2 = \sin^{-1}(C_{13}) \quad (1)
\]

\[
\theta_1 = \tan^{-1}\left(\frac{-C_{23}}{C_{33}}\right) \quad (2)
\]

\[
\theta_3 = \tan^{-1}\left(\frac{-C_{12}}{C_{11}}\right) \quad (3)
\]

Note that Eqs. (2) and (3) are singular when \(\cos(\theta_2) = 0\), that is, when \(\theta_2 = \frac{\pi}{2}\). In this case, we note

\[
C_{21} = C_{1} S_{3} + S_{1} C_{3} = \sin(\theta_1 + \theta_3) \quad (4)
\]

\[
C_{22} = C_{1} C_{3} - S_{1} S_{3} = \cos(\theta_1 + \theta_3) \quad (5)
\]

So, when \(\theta_2 = \frac{\pi}{2}\), we can only solve for the sum of the other two angles. Using Eqs. (4) and (5), we have

\[
\theta_1 + \theta_3 = \tan^{-1}\left(\frac{C_{21}}{C_{22}}\right).
\]