ME 6590 Multibody Dynamics
Coordinate Transformation Matrices Using Relative Coordinates

Instead of describing the orientation of a body relative to the inertial frame, it may be more convenient to describe its orientation relative to another body in the system. In such a situation, we can define coordinate transformation matrices from each body to the inertial frame and from one body to another.

Consider the two-body system shown with three reference frames: \( R : (N_1, N_2, N_3) \), \( B_1 : (e_1, e_2, e_3) \), and \( B_2 : (n_1, n_2, n_3) \). Using these frames, we can define four coordinate transformation matrices, \( [C_{B_1}] \), \( [C_{B_2}] \), \( [C_{B_1/B_1}] \), and \( [C_{B_2/B_1}] \). They are each defined by the following transformations

\[
\{N\} = [C_{B_1}] \{e\} \quad \{N\} = [C_{B_2}] \{n\} \quad \{e\} = [C_{B_1/B_1}] \{n\} \quad \{n\} = [C_{B_2/B_1}] \{e\}
\]

From the last two equations it is clear that \( [C_{B_2/B_1}]^T = [C_{B_2/B_2}] \). Also, using the first three equations, we can write

\[
\{N\} = [C_{B_1}] \{e\} = [C_{B_1}] [C_{B_2/B_1}] \{n\} = [C_{B_2}] \{n\}
\]

So, we conclude

\[
[C_{B_2}] = [C_{B_1}] [C_{B_2/B_1}]
\]

This result can be easily extended to include as many bodies as necessary to move from a body-fixed frame to the inertial frame through the frames of a series of interconnected bodies of the system.

\[
[C_{B_1}] = [C_{B_1}] [C_{B_2/B_1}] [C_{B_2/B_2}] \cdots [C_{B_{i+1}/B_i}]
\]