ME 6590 Multibody Dynamics  
Representation of Vector Operations as Matrix Operations

Dot (or Scalar) Product

Given two vectors
\[ a = a_1 e_1 + a_2 e_2 + a_3 e_3 \]
\[ b = b_1 e_1 + b_2 e_2 + b_3 e_3 \]
then the dot (or scalar) product of the two vectors is defined to be
\[ a \cdot b = \sum_{i=1}^{3} a_i b_i. \]

This product can be represented as the following matrix product
\[ a \cdot b \rightarrow \{a\}^T \{b\} \]
where \( \{a\} \) indicates a 3×1 column vector whose elements are the components of the vector \( a \), and \( \{a\}^T \) indicates a 1×3 row vector that is the transpose of \( \{a\} \). Recall that for any two vectors \( a \) and \( b \), \( a \cdot b = b \cdot a \). That is, the dot product is commutative. The matrix equivalent of this statement is \( \{a\}^T \{b\} = \{b\}^T \{a\} \).

Cross (or Vector) Product

Given two vectors
\[ a = a_1 e_1 + a_2 e_2 + a_3 e_3 \]
\[ b = b_1 e_1 + b_2 e_2 + b_3 e_3 \]
then the cross (or vector) product of the two vectors is defined to be
\[ a \times b = (a_2 b_3 - a_3 b_2)e_1 + (a_3 b_1 - a_1 b_3)e_2 + (a_1 b_2 - a_2 b_1)e_3. \]

This product can be represented as the following matrix product
\[ \mathbf{a} \times \mathbf{b} \rightarrow [\tilde{a}][b] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \]

where \([\tilde{a}]\) is a skew-symmetric matrix (defined above) containing the components of \(\mathbf{a}\). Recall that for any two vectors \(\mathbf{a}\) and \(\mathbf{b}\), \(\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}\). The matrix equivalent of this statement is \([\tilde{a}][b] = -[\tilde{b}][a]\).