Mass Center Velocities and Partial Velocities

Consider the rigid body shown in the diagram. Given that \((x_1, x_2, x_3)\) are the position coordinates of the mass center \(G\) relative to the inertial frame, then \(p_G\) the position vector of \(G\), \(\dot{r}_G\) the velocity of \(G\) in \(R\), and \([v_{G,\hat{x}}]\) the partial velocity matrix for \(G\) may be written in matrix form as

\[
\{p_G\} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \{v_G\} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}, \quad [v_{G,\hat{x}}] = \begin{bmatrix} \frac{\partial \dot{r}_G}{\partial \dot{x}_1} & \frac{\partial \dot{r}_G}{\partial \dot{x}_2} & \frac{\partial \dot{r}_G}{\partial \dot{x}_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

Velocities of Other Points

For other points in the system, such as point \(P\) in the diagram above, we can write the position vector in terms of the mass-center position vector as follows

\[
\{p_P\} = \{p_G\} + \{p_{P/G}\} = \{p_G\} + [C]\{p'_{P/G}\}
\]

Differentiating this expression gives the velocity of \(P\) as

\[
\{v_P\} = \{\dot{p}_P\} = \{\dot{p}_G\} + [\dot{C}]\{p'_{P/G}\} + [C]\{\dot{p}'_{P/G}\} = \{v_G\} + [\dot{C}]\{p'_{P/G}\}
\]

where \(\{\dot{p}'_{P/G}\} = \{0\}\) because \(\{p'_{P/G}\}\) represents the coordinates of \(P\) relative to \(G\) in the body-fixed system which are all constant.

To reduce Eq. (1) further, we must decide whether we will use inertial or body-fixed components for the angular velocity vectors. The two results are

\[
\{v_P\} = \{v_G\} + [\dot{\omega}_B][C]\{p'_{P/G}\}\\
\{v_P\} = \{v_G\} + [C][\dot{\omega}'_B]\{p'_{P/G}\}
\]

As before, the “primes” indicate components in the body-fixed system, and “no primes” indicate components in the inertial system.