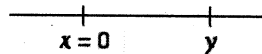
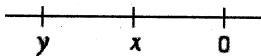
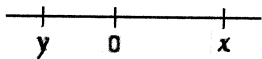


Triangle Inequality: $|x - y| \leq |x| + |y|$

Proof:

To see this, note that there are only three possibilities: x and y are on opposite sides of the number line; x and y are on the same side of the number line; or x and/or y are zero.



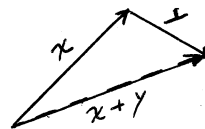
Recall that $|x - y|$ represents the distance from x to y . Thus $|x| = |x - 0|$ represents the distance from x to zero. Note that for each of the three illustrations, the distance between x and y is either smaller than or equal to the sum of the distance from x to zero and the distance from y to zero. That is $|x - y| \leq |x| + |y|$.

Alternate form 1: $|x + y| \leq |x| + |y|$

Proof:

$$|x + y| = |x - (-y)|$$

$$\leq |x| + |-y| = |x| + |y|$$



Alternate form 2: $|x| \leq |y| + |x - y|$

Proof:

$$|x| = |y + x - y| \leq |y| + |x - y|$$