

P218 · 16

Using Definition 2

$f(x) = \cos x$ We want to know $f'(x) = -\sin x$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x \cosh - \cos x}{h} - \frac{\sin x \sinh}{h} \\ &= \lim_{h \rightarrow 0} \cos x \left(\frac{\cosh - 1}{h} \right) - \sin x \left(\frac{\sinh}{h} \right) \\ &= \lim_{h \rightarrow 0} \cos x \cdot \lim_{h \rightarrow 0} \frac{\cosh - 1}{h} - \lim_{h \rightarrow 0} \sin x \cdot \lim_{h \rightarrow 0} \frac{\sinh}{h} \\ &= \cos x \times 0 - \sin x \times 1 \\ &= -\sin x \end{aligned}$$

$$\left. \begin{aligned} \lim_{h \rightarrow 0} \frac{\cosh - 1}{h} &= 0 \\ \lim_{h \rightarrow 0} \frac{\sinh}{h} &= 1 \end{aligned} \right\}$$