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Problem 57, page 289:-

Let the cubic function be: $f(x) = ax^3 + bx^2 + cx + d \rightarrow f'(x) = 3ax^2 + 2bx + c \rightarrow$

$$f''(x) = 6ax + 2b.$$

f Concave up when $6ax + 2b > 0$, $x > \frac{-b}{3a}$ concave down when $x < \frac{-b}{3a}$, the inflection occurs

when $x = \frac{-b}{3a}$. Because the graph has 3 x -intercepts, then the expression of $f(x)$ must factor as

$$f(x) = a(x - x_1)(x - x_2)(x - x_3)$$

$$= a[x^3 - (x_1 + x_2 + x_3)x^2 + (x_1x_2 + x_1x_3 + x_2x_3)x - x_1x_2x_3]$$

Equating the coefficients of the x^2 terms for the two forms of f gives us $b = -a(x_1 + x_2 + x_3)$, and

the x -coordinate of the inflection point is $\frac{-b}{3a} = \frac{-a(x_1 + x_2 + x_3)}{3a} = \frac{x_1 + x_2 + x_3}{3}$.