# Math 695: The Psychology of Learning Mathematics
# Fall 2000, Mondays 6:00 – 8:45 p.m.

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**Course Description:**

This course focuses on developing an understanding of what we know about how people think about mathematics and how an understanding of mathematics develops. The readings will provide an overview of various theoretical approaches used to better understand the teaching and learning of mathematics, with a focus on the K-12 level. The reading and writing assignments in this course will allow for insight into the existing evidence accumulated on teaching and learning mathematics and inspire thoughts for future directions in research.

**Course Responsibilities**:

The semester grade will be based on your class participation, including in the role of facilitator, and your writing assignments.

Class participation

A weekly assignment for the course is the reading and contemplation of the assigned readings. Since the format of the course is that of a discussion seminar, its success depends on your preparation and careful reading of the assignments, and then your participation in class discussions. Everyone should enter the classroom ready to discuss the required material in depth.

In addition, everyone will be asked to lead the class discussions three times during the semester. The first will be done with a partner on particular scholars in the field of cognitive psychology or mathematics education. The second will be on a topical area of research on learning in which you are interested. The third will be on chapters from the book, *Multiple Perspectives on Mathematics Teaching and Learning*. In the first two cases, you will be required to find readings by your scholar or on your topic to help prepare you for your role as facilitator and choose one or

two pertinent readings for your colleagues to read. You should also prepare for your colleagues a reference list for your scholar/topic and written summaries of the readings you have done. For your final experience facilitating discussion, you may choose to do additional readings and should prepare a list of discussion questions on the chapters from the book you are assigned. In every case, you are also required to meet with me before class (perhaps Thursday or Friday) to discuss your ideas/plans for your presentations/facilitation of discussion.

Writing Assignments

There will be three major writing assignments for the course:

**1) Learning Theorists**

Each of you will choose a particular scholar from the field of cognitive science or mathematics education to investigate in depth. You will then submit a paper, 7 to 10 pages in length, that includes the following:

(a) Historical background on the scholar—time period in which the scholar lived and important educational/societal issues of the time that impacted the scholar;
(b) Philosophical and/or theoretical beliefs about learning in general and/or in mathematics, specifically, and implications on teaching (again in general and/or in mathematics).

You should focus your discussion on the credibility of the ideas; whether the scholar provides evidence to support claims; whether the theory is useful; and whether the theory "rings true" based on your own personal experience. You should read "original" writings by the scholar as well as those written about the scholar by others in an attempt to "get to know" the scholar as well as you are able. Beyond submission of the paper, you will also present this information to the class with a handout that contains a detailed list of references on the scholar and a summary of what you have read. You should also choose one or two articles for your classmates to read to prepare for your presentation.

**2) Topical Research on Learning**

There are a multitude of research studies on the way students learn mathematics in the context of particular mathematical topics. It would be impossible to cover all of these studies individually, so each of you will be required to choose one topic to investigate in depth. You will then submit a "review of research" paper, 10-15 pages in length, that includes the following:

(a) summaries of research findings on the topic;
(b) critique of whether the data are believable and the research designs appropriate and effective;
(c) discussion of what the research findings suggest about the way children learn mathematics and implications for teaching and whether these ideas are discussed explicitly or implicitly in the articles;
(d) suggestions for further research on the learning of the topic.

Topics to choose from include algebra, functions, geometry, problem solving, proof, proportional reasoning, rational numbers, and whole number computation. Beyond submission of the paper, you will also present this information to the class with a handout that contains a detailed list of references and a summary of what you have read. You should also choose one or two articles for your classmates to read to prepare for your presentation.

**3) Synthesis of Learning Theories and Implications for Teaching**

The objective of this assignment is to encourage reconsideration and reflection on the major themes of the course in light of the most current movement in situated cognition. This assignment will be in the format of a "practice" preliminary exam (take-home) and will provide a series of questions to which you will respond. Further details will be provided later.

There may also be additional short writing assignments throughout the semester if deemed necessary.

## TENTATIVE SCHEDULE

**Aug. 28** **Current Vision of Mathematics Teaching and Learning**

National Council of Teachers of Mathematics. (2000). Teaching and Learning Principles, Principles and Standards for School Mathematics, pp. 16-21.

**Implications of Learning Theories on Teaching**

Maher, Carolyn and Davis, Robert. (1990). Building Representations of Children's Meaning in Davis, R., Maher, C. & Noddings, N. (Eds.) Constructivist Views on the Teaching and Learning of Mathematics, JRME Monograph No. 4, pp. 79-90.

**Sep. 11** **Mathematics as Computation**

Resnick, L.B. & Ford, W.W. (1981) Psychology of Mathematics for Instruction. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc. [Chapters 1 and 2].

**Original Writings**

Thorndike, E. L. (1920). The psychology of drill in arithmetic: The strength of bonds. (Selected portion reprinted in J. K. Bidwell and R. G. Clason (Eds.) Readings in the history of mathematics education, pp.154-162). Washington, DC: National Council of Teachers of Mathematics.

Skinner, B. F. (1969). The science of learning and the art of teaching, The technology of teaching (pp. 9-28). Englewood Cliffs, NJ: Prentice-Hall, Inc.

**Mathematics as Conceptual Understanding and Problem Solving**

Resnick, L.B. & Ford, W.W. (1981) Psychology of Mathematics for Instruction. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc. [Chapters 5 and 6].

**Original Writings**

McLellan, J. A., & Dewey, J. (1895). The psychology of number. (Selected portion reprinted in J. K. Bidwell & R. G. Clason (eds.) Readings in the history of mathematics education, pp.154-162). Washington, D. C.: NCTM.

Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In W. D. Reeve (Ed.), The teaching of arithmetic (Tenth Yearbook of the National Council of Teachers of Mathematics) (pp. 1-31). New York: Columbia University, Teachers College, Bureau of Publications.

**Sep. 18** **Learning Theorists** (Readings to be assigned by presenters)

Johann Pestalozzi/Warren Colburn

Jerome Bruner/William Brownell

**Sep. 25** **Learning Theorists, continued** (Readings to be assigned by presenter)

Piaget

**Mathematics as Conceptual Understanding by Information-Processing Theorists**

Siegler, Robert (1998). Children's Thinking. Information-Processing Theories of Development, Chapter 3, pp. 63-100.

**Original Writings**

Case, R. and Sandieson, R. (1988). A Developmental approach to the identification and teaching of central conceptual structures in mathematics and science in the middle grades. In Hiebert, J. and Behr, M. (Eds.) Number Concepts and Operations in the Middle Grades. Reston, VA: NCTM.

**Oct. 2** **Results of Traditional Instruction/Misconceptions and "Bugs"**

Brown, J. S., & VanLehn, K. (1982). Towards a generative theory of "bugs". In T. P. Carpenter, J. M. Moser, & T. A. Romberg (Eds.), Addition and subtraction: A cognitive perspective, (pp. 117-135). Hillsdale, NJ: Erlbaum.

Erlwanger, S.H. (1975). Case studies of children's conceptions of mathematics - Part I. Journal of Children's Mathematical Behavior, 1(3), 157-283. (Only the first case is included here, pp 157-232).

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra (pp. 60-86). Reston, VA: The Council.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well taught" mathematics courses. Educational Psychologist, 23, 145-166.

**Oct. 9** **Procedural and Conceptual Knowledge**

Hiebert, J.. (Ed.) (1986). Conceptual and procedural knowledge: The case of mathematics. Hillsdale, NJ: Erlbaum. Chapters to be announced.

**Oct. 16** **Teaching and Learning Mathematics for Understanding**

Hiebert, J. & Carpenter, T. P. (1992). Learning and Teaching with Understanding. In D. A. Grouws, ed, Handbook of research on mathematics teaching and learning, p. 65-100. New York, NY: Macmillian Publishing Company. [Chapter 4]

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. Journal for Research in Mathematics Education, 24(1), 8-40.

**Topical Research Studies** (Readings to be assigned by presenters)

Topics include algebra, functions, geometry, problem solving, proof, proportional reasoning, rational numbers, and whole number computation.

**Oct. 23** **Topical Research Studies, continued** (Readings to be assigned by presenters)

**Oct. 30** **Constructivism and Social Aspects of Learning**

Davis, R. B., Maher, C. A., & N. Noddings, N. (Eds.) (1990). Constructivist views on the teaching and learning of mathematics, Monograph No. 4. Reston, VA: National Council of Teachers of Mathematics. [Chapters 1-3, pp. 1-47]

Phillips, D. C. (1995). The good, the bad, and the ugly: The many faces of constructivism. Educational Researcher, 24(7), 5-12.

von Glaserfeld, E. (1996). Footnotes to "The many faces of constructivism" [Response to The good, the bad, and the ugly: The many faces of constructivism]. Educational Researcher, 25(6), 19.

Davydov, V. (1995). The influence of L. S. Vygotsky on education theory, research, and practice. Educational Researcher, 24(3), 12-21.

Karpov, Y. V. & Bransford, J. D. (1995). L. S. Vygotsky and the doctorine of empirical and theoretical learning. Educational Psychologist, 30, 61-66.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. American Educational Research Journal, 23(7), 13-20. **OR**

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of the mind in mathematics education. Journal for Research in Mathematics Education, 23(1), 2-33.

**Nov. 6** **Situated Cognition and Problem Solving**

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. Educational Researcher, 18, 32-42.

Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated learning and education. Educational Researcher, 25(4), 5-11.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Oliver, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. Educational Researcher, 25(4), 12-21.[Intro by Donmoyer, p.4]

Anderson, J. R., Reyder, L. M., & Simon, H. A. (1997). Situated versus cognitive perspectives: Form versus substance. Educational Researcher, 26(1), 18-21.

Kirshner, D., & Whitson, J. A. (1998). Obstacles to Understanding Cognition as Situated. Educational Researcher, 27(8), 22-28. [Response to Anderson, Reder & Simon, 1996]

**Nov. 13** **Multiple Perspectives on Mathematics Teaching and Learning**
Chapters 1-4

**Nov. 20** **Multiple Perspectives on Mathematics Teaching and Learning**
Chapters 5-8

**Nov. 27** **Multiple Perspectives on Mathematics Teaching and Learning**
Chapters 9 & 10

**Synthesis of Ideas on Learning Mathematics and Implications for Teaching/Thoughts for the Future**

## ADDITIONAL READINGS

**Overviews of Theories of Learning**


Steffe, L. P., Nesher, P., Cobb, P., Goldin, G. Q. & Greer, B. (1996) Theories of Mathematical Learning. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

- Carpenter, T. P. (1980). Research in Cognitive Development. In R. J. Shumway (Ed.), Research in mathematics education (pp. 146-206). Reston, VA: The National Council of Teachers of Mathematics.

Schoenfeld, A.H. (1987). Cognitive science in mathematics education: An overview. In A.H. Schoenfeld (Ed.), Cognitive Science and Mathematics Education (pp 1-32). Hillsdale, NJ: Erlbaum.

Kieran, C.A. (1994). Doing and seeing things differently: A 25-Year retrospective of mathematics education research on learning. Journal for Research in Mathematics Education, 25(6), 583-607.


**Historical Perspectives: Original Writings**


Ausubel, D. P. (1968). Educational psychology: A cognitive view. (pp. vi, 83-89, 107-115, 136-138). New York: Holt, Rinehart, and Winston, Inc.

Gagne, R. M. (1965). Learning hierarchies. In Conditions of learning (pp. 237-257). New York: Holt, Rinehart, and Winston, Inc.

- Skemp. R. R. (1987). The psychology of learning mathematics. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Van Engen, H. (1949). An analysis of meaning in arithmetic. Elementary School Journal, 49, 321-329, 395-400.


**Implications for Teaching**


Carpenter, T. P., Hiebert, J., & Moser, J. M. (1983). The effect of instruction on children's solutions of addition and subtraction word problems. Educational Studies in Mathematics, 14, 55-72.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics. Journal for Research in Mathematics Education, 27(4), 403-434.

Cobb, P. (1990). Multiple perspectives. In L. Steffe & T. Wood (Eds.), Transforming children's mathematics education, (pp. 200-215). Hillsdale, NJ: Erlbaum.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. Cognition and Instruction, 3, 305-342.

Wearne, D., & Hiebert, J. (1988). A cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers. Journal for Research in Mathematics Education, 19, 371-384.


**Constructivism**


Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal For Research in Mathematics Education, 26(2), 114-145.

Steffe, L. P. & Kieren, T. (1994). Radical constructivism and mathematics education. Journal for Research in Mathematics Education, 25(6), 711-733.

Steffe, L. P. & D'Ambrosio, B. S. (1995). Toward a working model of constructivist teaching: A reaction to Simon. Journal For Research in Mathematics Education, 26(2), 146-159.

Simon, M. A. (1995). Elaborating models of mathematics teaching: A response to Steffe and D'Ambrosio. Journal For Research in Mathematics Education, 26(2), 160-164.

Kamii, C. (1985). The importance of social interaction, Young children reinvent arithmetic, (pp. 26-70). New York: Teachers College Press.

**Discourse**

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic classrooms. American Educational Research Journal, 30(2), 393-425.

Cobb, P., Wood, T., Yackel, E., & McNeal, L. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. American Educational Research Journal, 29(3), 573-604.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. Cognition and Instruction, 3, 305-342.

Forman, E. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 115-129). Mahwah, NJ: Lawrence Erlbaum Associates.

Cazden, C. B. (1988) The structure of lessons. In C. B. Cazden, Classroom Discourse: The Language of Teaching and Learning (pp. 29-52). Portsmouth, NH: Heinemann.

**Elementary Grades**

El'konin, D. B. (1975). Primary schoolchildren's intellectual capabilities and the content of instruction. In L. P. Steffe (Ed.), Soviet studies in the psychology of learning and teaching mathematics, (Vol. 7, pp. 13-54). Palo Alto, CA: Stanford University, School Mathematics Study Group.

Hiebert, J., & Carpenter, T. P. (1982). Piagetian tasks as readiness measures in math instruction: A critical review. Educational Studies in Mathematics, 13, 329-345.

Piaget, J. (1964). Development and learning. Journal for Research in Science Teaching, 2, 176-186.

**Early Number & Arithmetic Abilities**

Carpenter, T.P., Ansell, E., Franke, M.L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. Journal for Research in Mathematics Education, 24(5), 428-441.

Riley, M.S., Greeno, J.G., & Heller, J.I. (1983). Development of children's problem-solving ability in arithmetic. In H.P. Ginsburg (Ed.), The development of mathematical thinking (pp. 153-196). New York: Academic Press.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction of concepts in grades one through three. Journal for Research in Mathematics Education, 15(3), 179-202.

**Everyday Arithmetic Abilities**

Saxe, G.B. (1988). Candy selling and math learning. Educational Researcher, 17(6), 14-21.

Lave, J., Murtaugh, M., & de la Rocha, O. (1984). The dialectical construction of arithmetic in grocery shopping. In B. Rogoff & J. Lave (Eds.), Everyday cognition: Its development in social context (pp. 67-94). Cambridge, MA: Harvard University.

Carraher, T.N., Schliemann, A.D., & Carraher, D.W. (1988). Mathematical concepts in everyday life. In G.B. Saxe & M. Gearhart (Eds.), Children's mathematics (pp. 71-87). San Francisco: Jossey-Bass.

**Problem Solving**

Overviews:

Lester, F. (1994.) Musings about mathematical problem-solving research: 1970-1994. Journal for Research in Mathematics Education, 25 (6), pp. 660-675.

Silver, E. (Ed.) (1985). Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates.

(Both sources list research studies in references.)

**The Middle Grades & Beyond**

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), Number Concepts and Operations in the Middle Grades (pp.211-228). Reston, VA: The Council.

Foltz, C., Overton, W. F., & Ricco, R. B. (1995). Proof construction: Adolescent development from inductive to deductive problem-solving strategies. Journal of Experimental Child Psychology, 59 , 179-195.

Hart, K. (1988). Ratio and proportion. In J. Hiebert & M. Behr (Eds.), Number concepts and operations in the middle grades, (pp. 198-219). Reston, VA: National Council of Teachers of Mathematics.

Mack, N. K. (1993). Learning rational numbers with understanding: The case of informal knowledge. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.) Rational numbers: An integration of research (pp. 85-105). Hillsdale, NJ: Erlbaum.

Wearne, D., & Hiebert, J. (1988). A cognitive approach to meaningful mathematics instruction: Testing a local theory using decimal numbers. Journal for Research in Mathematics Education, 19, 371-384.

Hiebert, J. & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal and number knowledge. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics (pp. 199-223). Hillsdale, NJ: Erlbaum.

Ohlsson, S. (1988). Mathematical meaning and applicational meaning in the semantics of fractions and related concepts. In J. Hiebert & M. Behr (Eds.), Number concepts and operations in the middle grades, (pp. 53-92). Reston, VA: National Council of Teachers of Mathematics.

**The High School Level & Beyond**

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.), Research issues in the learning and teaching of algebra, (pp. 60-86). Reston, VA: National Council of Teachers of Mathematics.

Matz, M. (1980). Towards a computational theory of algebraic competence. Journal of Mathematical Behavior, 3(1), 93-166.

Linchevski, L. & Herscovics, N. (1996). Crossing the Cognitive Gap between Arithmetic and Algebra: Operating on the Unknown in the Context of Equations. Educational Studies in Mathematics, 30(1), 39-65.

Tall, D. & Thomas, M. (1991). Encouraging Versatile Thinking in Algebra Using the Computer. Educational Studies in Mathematics, 22(2), 125-147.

Resnick, L. B., Cauzinille-Marmeche, E., & Mathieu, J. (1987). Understanding algebra. In J. Sloboda & D. Rogers (Eds.), Cognitive processes in mathematics, (pp. 169-225). Oxford: Clarendon Press.

Tall, David. (1990). Inconsistencies in the Learning of Calculus and Analysis. Focus on Learning Problems in Mathematics, 12(3-4), 49-63.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. Journal for Research in Mathematics Education, 19(1), 3-25.

White, P., & Mitchelmore, M. (1996). Conceptual knowledge in introductory calculus. Journal for Research in Mathematics Education, 27(1), 79-95.

Batanero, C., Estepa, A., Godino, J., & Green, D. R. (1996). Intuitive strategies and preconceptions about association in contingency tables. Journal for Research in Mathematics Education, 27(2), 151-169.