Atmospheric Optics

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Abstract
Colors of the sky and colored displays in the sky are mostly a consequence of selective scattering by molecules or particles, absorption usually being irrelevant. Molecular scattering selective by wavelength – incident sunlight of some wavelengths being scattered more than others – but the same in any direction at all wavelengths gives rise to the blue of the sky and the red of sunsets and sunrises. Scattering by particles selective by direction – different in different directions at a given wavelength – gives rise to rainbows, coronas, iridescent clouds, the glory, sun dogs, halos, and other ice-crystal displays. The size distribution of these particles and their shapes determine what is observed, water droplets and ice crystals, for example, resulting in distinct displays.

To understand the variation and color and brightness of the sky as well as the brightness of clouds requires coming to grips with multiple scattering: scatterers in an ensemble are illuminated by incident sunlight and by the scattered light from each other. The optical properties of an ensemble are not necessarily those of its individual members.

Mirages are a consequence of the spatial variation of coherent scattering (refraction) by air molecules, whereas the green flash owes its existence to both coherent scattering by molecules and incoherent scattering by molecules and particles.

Keywords
sky colors; mirages; green flash; coronas; rainbows; the glory; sun dogs; halos; visibility.

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1
Introduction

Atmospheric optics is nearly synonymous with light scattering, the only restrictions being that the scatterers inhabit the atmosphere and the primary source of their illumination is the sun. Essentially all light we see is scattered light, even that directly from the sun. When we say that such light is unscattered we really mean that it is scattered in the forward direction;

hence it is *as if* it were unscattered. Scattered light is radiation from matter excited by an external source. When the source vanishes, so does the scattered light, as distinguished from light emitted by matter, which persists in the absence of external sources.

Atmospheric scatterers are either molecules or particles. A particle is an aggregation of sufficiently many molecules that it can be ascribed macroscopic properties such as temperature and refractive index. There is no canonical number of molecules that must unite to form a *bona fide* particle. Two molecules clearly do not a quorum make, but what about 10, 100, 1000? The particle size corresponding to the largest of these numbers is about 10^{-3} μm. Particles this small of water substance would evaporate so rapidly that they could not exist long under conditions normally found in the atmosphere. As a practical matter, therefore, we need not worry unduly about scatterers in the shadow region between molecule and particle.

A property of great relevance to scattering problems is *coherence*, both of the array of scatterers and of the incident light. At visible wavelengths, air is an array of incoherent scatterers: the radiant power scattered by N molecules is N times that scattered by one (except in the forward direction). But when water vapor in air condenses, an incoherent array is transformed into a coherent array: uncorrelated water molecules become part of a single entity. Although a single droplet is a coherent array, a cloud of droplets taken together is incoherent.

Sunlight is incoherent but not in an absolute sense. Its lateral coherence length is tens of micrometers, which is why we can observe what are essentially interference patterns (e.g., coronas and glories) resulting from illumination of cloud droplets by sunlight.

This article begins with the color and brightness of a purely molecular atmosphere, including their variation across the vault of the sky. This naturally leads to the state of polarization of skylight. Because the atmosphere is rarely, if ever, entirely free of particles, the general characteristics of scattering by particles follow, setting the stage for a discussion of atmospheric visibility.

Atmospheric refraction usually sits by itself, unjustly isolated from all those atmospheric phenomena embraced by the term scattering. Yet refraction is another manifestation of scattering, coherent scattering in the sense that phase differences cannot be ignored.

Scattering by single water droplets and ice crystals, each discussed in turn, yields feasts for the eye as well as the mind. The curtain closes on the optical properties of clouds.

2 Color and Brightness of Molecular Atmosphere

2.1 A Brief History

Edward Nichols began his 1908 presidential address to the New York meeting of the American Physical Society as follows: ‘‘In asking your attention to-day, even briefly, to the consideration of the present state of our knowledge concerning the color of the sky it may be truly said that I am inviting you to leave the thronged thoroughfares of our science for some quiet side street where little is going on and you may even suspect that I am coaxing you into some blind alley, the inhabitants of which belong to the dead past.''

Despite this depreciatory statement, hoary with age, correct and complete explanations of the color of the sky still are hard to find. Indeed, all the faulty explanations lead active lives: the blue sky is the reflection of the blue sea; it is caused by water, either vapor or droplets or both; it is caused by dust. The true cause of the blue sky is not difficult to understand, requiring only a bit of critical thought stimulated by belief in the inherent fascination of all natural phenomena, even those made familiar by everyday occurrence.

Our contemplative prehistoric ancestors no doubt speculated on the origin of the blue sky, their musings having vanished into it. Yet it is curious that Aristotle, the most prolific speculator of early recorded history, makes no mention of it in his *Meteorologica* even though he delivered pronouncements on rainbows, halos, and mock suns and realized that ''the sun looks red when seen through mist or smoke.'' Historical discussions of the blue sky sometimes cite Leonardo as the first to comment intelligently on the blue of the sky, although this reflects a European bias. If history were to be written by a supremely disinterested observer, Arab philosophers would likely be given more credit for having had profound insights into the workings of nature many centuries before their European counterparts descended from the trees. Indeed, Möller [1] begins his brief history of the blue sky with Jakub Ibn Ishak Al Kindi (800–870), who explained it as ''a mixture of the darkness of the night with the light of the dust and haze particles in the air illuminated by the sun.''

Leonardo was a keen observer of light in nature even if his explanations sometimes fell short of the mark. Yet his hypothesis that ''the blueness we see in the atmosphere is not intrinsic color, but is caused by warm vapor evaporated in minute and insensible atoms on which the solar rays fall, rendering them luminous against the infinite darkness of the fiery sphere which lies beyond and includes it'' would, with minor changes, stand critical scrutiny today. If we set aside Leonardo as *sui generis*, scientific attempts to unravel the origins of the blue sky may be said to have begun with Newton, that towering pioneer of optics, who, in time-honored fashion, reduced it to what he already had considered: interference colors in thin films. Almost two centuries elapsed before more pieces in the puzzle were contributed by the experimental investigations of von Brücke and Tyndall on light scattering by suspensions of particles. Around the same time Clausius added his bit in the form of a theory that scattering by minute bubbles causes the blueness of the sky. A better theory was not long in coming. It is associated with a man known to the world as Lord Rayleigh even though he was born John William Strutt.

Rayleigh's paper of 1871 marks the beginning of a satisfactory explanation of the blue sky. His scattering law, the key to the blue sky, is perhaps the most famous result ever obtained by dimensional analysis. Rayleigh argued that the field E_s scattered by a particle small compared with the light illuminating it is proportional to its volume V and to the incident field E_i. Radiant energy conservation requires that the scattered field diminish inversely as the distance r from the particle so that the scattered power diminishes as the square of r. To make this proportionality dimensionally homogeneous requires the inverse square of a quantity with the dimensions of length. The only plausible physical variable at hand is the wavelength of the incident light, which leads to

$$E_s \propto \frac{E_i V}{r\lambda^2}. \qquad (1)$$

When the field is squared to obtain the scattered power, the result is Rayleigh's inverse fourth-power law. This law is really only an often – but not always – very good approximation. Missing from it are dimensionless properties of the particle such as its refractive index, which itself depends on wavelength. Because of this *dispersion*, therefore, nothing scatters exactly as the inverse fourth power.

Rayleigh's 1871 paper did not give the complete explanation of the color and polarization of skylight. What he did that was not done by his predecessors was to give a law of scattering, which could be used to test quantitatively the hypothesis that selective scattering by atmospheric particles could transform white sunlight into blue skylight. But as far as giving the agent responsible for the blue sky is concerned, Rayleigh did not go essentially beyond Newton and Tyndall, who invoked particles. Rayleigh was circumspect about the nature of these particles, settling on salt as the most likely candidate. It was not until 1899 that he published the capstone to his work on skylight, arguing that air molecules themselves were the source of the blue sky. Tyndall cannot be given the credit for this because he considered air to be *optically empty*: when purged of all particles it scatters no light. This erroneous conclusion was a result of the small scale of his laboratory experiments. On the scale of the atmosphere, sufficient light is scattered by air molecules to be readily observable.

2.2 Molecular Scattering and the Blue of the Sky

Our illustrious predecessors all gave explanations of the blue sky requiring the presence of water in the atmosphere: Leonardo's "evaporated warm vapor," Newton's "Globules of water," Clausius's bubbles. Small wonder, then, that water still is invoked as the cause of the blue sky. Yet a cause of something is that without which it would not occur, and the sky would be no less blue if the atmosphere were free of water.

A possible physical reason for attributing the blue sky to water vapor is that, because of selective *absorption*, liquid water (and ice) is blue upon transmission of white light over distances of order meters. Yet if all the water in the atmosphere at any instant were to be compressed into a liquid, the result would be a layer about 1 cm thick, which is not sufficient to transform white light into blue by selective absorption.

Water vapor does not compensate for its hundredfold lower abundance than nitrogen and oxygen by greater scattering per molecule. Indeed, scattering of visible light by a water molecule is slightly *less* than that by either nitrogen or oxygen.

Scattering by atmospheric molecules does not obey Rayleigh's inverse fourth-power law exactly. A least-squares fit over the visible spectrum from 400 to 700 nm of the *molecular scattering coefficient* of sea-level air tabulated by Penndorf [2] yields an inverse 4.089th-power scattering law.

The molecular scattering coefficient β, which plays important roles in following sections, may be written

$$\beta = N\sigma_s, \qquad (2)$$

where N is the number of molecules per unit volume and σ_s, the scattering cross section (an average because air is a mixture) per molecule, approximately obeys Rayleigh's law. The form of this expression betrays the incoherence of scattering by atmospheric molecules. The inverse of β is interpreted as the scattering *mean free path*, the average distance a photon must travel before being scattered.

To say that the sky is blue because of Rayleigh scattering, as is sometimes done, is to confuse an agent with a law. Moreover, as Young [3] pointed out, the term Rayleigh scattering has many meanings. Particles small compared with the wavelength scatter according to the same law as do molecules. Both can be said to be Rayleigh scatterers, but only molecules are necessary for the blue sky. Particles, even small ones, generally diminish the vividness of the blue sky.

Fluctuations are sometimes trumpeted as the "real" cause of the blue sky. Presumably, this stems from the fluctuation theory of light scattering by media in which the scatterers are separated by distances small compared with the wavelength. In this theory, which is associated with Einstein and Smoluchowski, matter is taken to be continuous but characterized by a refractive index that is a random function of position. Einstein [4] stated that "it is remarkable that our theory does not make *direct* use of the assumption of a discrete distribution of matter." That is, he circumvented a difficulty but realized it could have been met head on, as Zimm [5] did years later.

The blue sky is really caused by scattering by molecules – to be more precise, scattering by bound electrons: free electrons do not scatter selectively. Because air molecules are separated by distances small compared with the wavelengths of visible light, it is not obvious that the power scattered by such molecules can be added. Yet if they are completely uncorrelated, as in an ideal gas (to good approximation the atmosphere is an ideal gas), scattering by N molecules is N times scattering by one. This is the only sense in which the blue sky can be attributed to scattering by fluctuations. Perfectly homogeneous matter does not exist. As stated pithily by Planck, "a chemically pure substance may be spoken of as a vacuum made turbid by the presence of molecules."

2.3 Spectrum and Color of Skylight

What is the spectrum of skylight? What is its color? These are two different questions. Answering the first answers the second but not the reverse. Knowing the color of skylight we cannot uniquely determine its spectrum because of *metamerism*: A given perceived color can in general be obtained in an indefinite number of ways.

Skylight is not blue (itself an imprecise term) in an absolute sense. When the visible spectrum of sunlight outside the earth's atmosphere is modulated by Rayleigh's scattering law, the result is a spectrum of scattered light that is neither solely blue nor even peaked in the blue (Fig. 1). Although blue does not predominate spectrally, it does predominate perceptually. We perceive the sky to be blue even though skylight contains light of all wavelengths.

Any source of light may be looked upon as a mixture of white light and light of a single wavelength called the *dominant wavelength*. The *purity* of the source is the relative amount of the monochromatic component in the mixture. The dominant wavelength of sunlight scattered according to Rayleigh's law is about 475 nm, which lies solidly in the blue if we take this to mean light with wavelengths between 450 and 490 nm. The purity of this

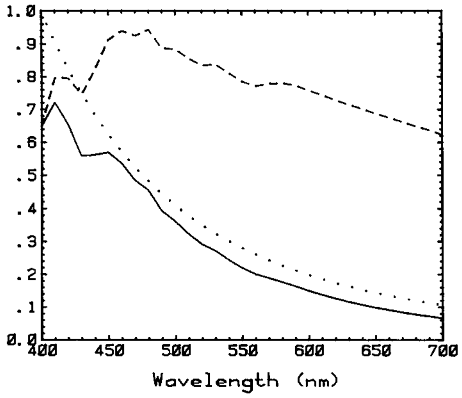

Fig. 1 Rayleigh's scattering law (dots), the spectrum of sunlight outside the Earth's atmosphere (dashes), and the product of the two (solid curve). The solar spectrum is taken from Thekaekara, M. P., Drummond, A. J. (**1971**), *Nat. Phys. Sci.* **229**, 6–9 [6]

scattered light, about 42%, is the upper limit for skylight. Blues of real skies are less pure.

Another way of conveying the color of a source of light is by its *color temperature*, the temperature of a blackbody having the same perceived color as the source. Since blackbodies do not span the entire gamut of colors, not all sources of light can be assigned color temperatures. But many natural sources of light can. The color temperature of light scattered according to Rayleigh's law is infinite. This follows from Planck's spectral emission function $e_{b\lambda}$ in the limit of high temperature,

$$e_{b\lambda} \approx \frac{2\pi ckT}{\lambda^4}, \quad \frac{hc}{\lambda} \ll kT, \tag{3}$$

where h is Planck's constant, k is Boltzmann's constant, c is the speed of light *in vacuo*, and T is absolute temperature. Thus, the emission spectrum of a blackbody with an infinite temperature has the same functional form as Rayleigh's scattering law.

2.4 Variation of Sky Color and Brightness

Not only is skylight not pure blue, but its color and brightness vary across the vault of the sky, with the best blues at zenith. Near the astronomical horizon the sky is brighter than overhead but of considerably lower purity. That this variation can be observed from an airplane flying at 10 km, well above most particles, suggests that the sky is inherently nonuniform in color and brightness (Fig. 2). To understand why requires invoking multiple scattering.

Multiple scattering gives rise to observable phenomena that cannot be explained solely by single-scattering arguments. This is easily demonstrated. Fill a blackened pan

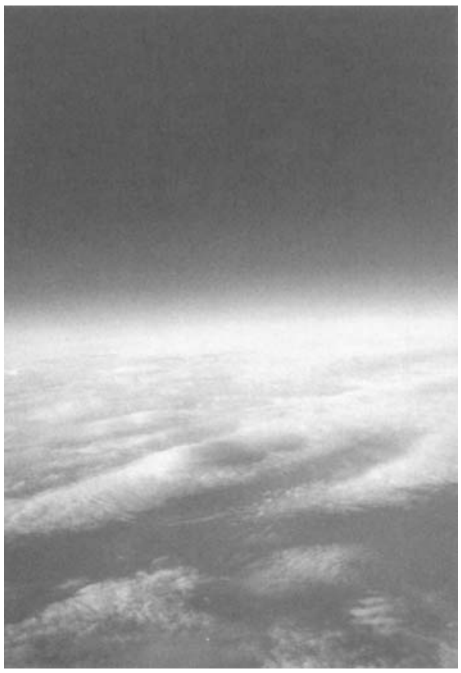

Fig. 2 Even at an altitude of 10 km, well above most particles, the sky brightness increases markedly from the zenith to the astronomical horizon

with clean water, then add a few drops of milk. The resulting dilute suspension illuminated by sunlight has a bluish cast. But when more milk is added, the suspension turns white. Yet the properties of the scatterers (fat globules) have not changed, only their *optical thickness*: the blue suspension being optically thin, the white being optically thick.

Optical thickness is physical thickness in units of scattering mean free path, and hence is dimensionless. The optical thickness τ between any two points connected by an arbitrary path in a medium populated by (incoherent) scatterers is an integral over the path:

$$\tau = \int_1^2 \beta \, \mathrm{d}s. \tag{4}$$

The *normal optical thickness* τ_n of the atmosphere is that along a radial path extending from the surface of the Earth to infinity. Figure 3 shows τ_n over the visible spectrum for a purely molecular atmosphere. Because τ_n is generally small compared with unity, a photon from the sun traversing a radial path in the atmosphere is unlikely to be scattered more than once. But along a tangential path, the optical thickness is about 35 times greater (Fig. 4), which leads to several observable phenomena.

Even an intrinsically black object is luminous to an observer because of *airlight*, light scattered by all the molecules and particles along the line of sight from observer to object. Provided that this is uniformly illuminated by sunlight and that ground reflection is negligible, the airlight radiance L is approximately

$$L = GL_0(1 - \mathrm{e}^{-\tau}), \tag{5}$$

where L_0 is the radiance of incident sunlight along the line of sight with optical thickness τ. The term G accounts for geometric reduction of radiance because of scattering of nearly monodirectional sunlight in all directions. If the line of sight is uniform in composition, $\tau = \beta d$, where β is the scattering coefficient and d is the physical distance to the black object.

If τ is small ($\ll 1$), $L \approx GL_0\tau$. In a purely molecular atmosphere, τ varies with wavelength according to Rayleigh's law; hence the distant black object in such an atmosphere is perceived to be

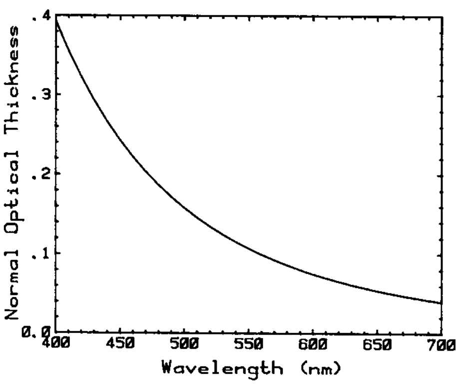

Fig. 3 Normal optical thickness of a pure molecular atmosphere

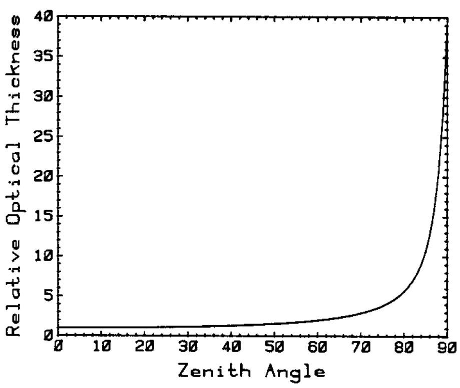

Fig. 4 Optical thickness (relative to the normal optical thickness) of a molecular atmosphere along various paths with zenith angles between 0° (normal) and 90° (tangential)

bluish. As τ increases so does L but not proportionally. Its limit is GL_0: The airlight radiance spectrum is that of the source of illumination. Only in the limit $d = 0$ is $L = 0$ and the black object truly black.

Variation of the brightness and color of dark objects with distance was called *aerial perspective* by Leonardo. By means of it we estimate distances to objects of unknown size such as mountains.

Aerial perspective belongs to the same family as the variation of color and brightness of the sky with zenith angle. Although the optical thickness along a path tangent to the Earth is not infinite, it is sufficiently large (Figs. 3 and 4) that GL_0 is a good approximation for the radiance of the horizon sky. For isotropic scattering (a condition almost satisfied by molecules), G is around 10^{-5}, the ratio of the solid angle subtended by the sun to the solid angle of all directions (4π). Thus, the horizon sky is not nearly so bright as direct sunlight.

Unlike in the milk experiment, what is observed when looking at the horizon sky is not multiply scattered light. Both have their origins in multiple scattering but manifested in different ways. Milk is white because it is weakly absorbing and optically thick, and hence all components of incident white light are multiply scattered to the observer even though the blue component traverses a shorter average path in the suspension than the red component. White horizon light has escaped being multiply scattered, although multiple scattering is why this light is white (strictly, has the spectrum of the source). More light at the short-wavelength end of the spectrum is scattered *toward* the observer than at the long-wavelength end. But long-wavelength light has the greater likelihood of being transmitted to the observer without being scattered *out of* the line of sight. For a long optical path, these two processes compensate, resulting in a horizon radiance spectrum which is that of the source.

Selective scattering by molecules is not sufficient for a blue sky. The atmosphere also must be optically thin, at least for most zenith angles (Fig. 4) (the blackness of space as a backdrop is taken for granted but also is necessary, as Leonardo recognized). A corollary of this is that the blue sky is not inevitable: an atmosphere composed entirely of nonabsorbing, selectively scattering molecules overlying a nonselectively reflecting earth need not be blue. Figure 5 shows calculated spectra of the zenith sky over black ground for a molecular atmosphere with the present normal optical thickness as well as for hypothetical atmospheres 10 and 40 times thicker. What we take to be inevitable is accidental: If our atmosphere were much thicker, but identical in composition, the color of the sky would be quite different from what it is now.

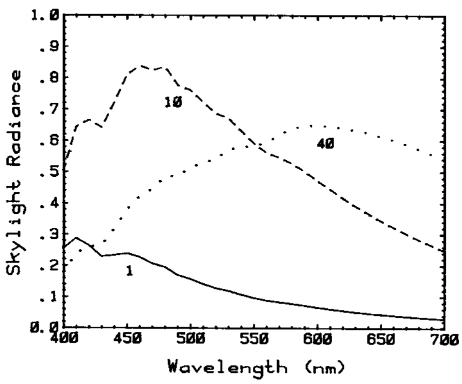

Fig. 5 Spectrum of overhead skylight for the present molecular atmosphere (solid curve), as well as for hypothetical atmospheres 10 (dashes) and 40 (dots) times thicker

2.5
Sunrise and Sunset

If short-wavelength light is preferentially scattered out of direct sunlight, long-wavelength light is preferentially transmitted in the direction of sunlight. Transmission is described by an exponential law (if light multiply scattered back into the direction of the sunlight is negligible):

$$L = L_0 e^{-\tau}, \tag{6}$$

where L is the radiance at the observer in the direction of the sun, L_0 is the radiance of sunlight outside the atmosphere, and τ is the optical thickness along this path.

If the wavelength dependence of τ is given by Rayleigh's law, sunlight is *reddened* upon transmission: The spectrum of the transmitted light is comparatively richer than the incident spectrum in light at the long-wavelength end of the visible spectrum. But to say that transmitted sunlight is reddened is not the same as saying it is red. The perceived color can be yellow, orange, or red, depending on the magnitude of the optical thickness. In a molecular atmosphere, the optical thickness along a path from the sun, even on or below the horizon, is not sufficient to give red light upon transmission. Although selective scattering by molecules yields a blue sky, reds are not possible in a molecular atmosphere, only yellows and oranges. This can be observed on clear days, when the horizon sky at sunset becomes successively tinged with yellow, then orange, but not red.

Equation (6) applies to the radiance only in the direction of the sun. Oranges and reds can be seen in other directions because reddened sunlight illuminates scatterers not lying along the line of sight to the sun. A striking example of this is a horizon sky tinged with oranges and pinks in the direction *opposite* the sun.

The color and brightness of the sun changes as it arcs across the sky because the optical thickness along the line of sight changes with solar zenith angle Θ. If the Earth were flat (as some still aver), the transmitted solar radiance would be

$$L = L_0 e^{\tau_n / \cos\Theta}. \tag{7}$$

This equation is a good approximation except near the horizon. On a flat earth, the optical thickness is infinite for horizon paths. On a spherical earth, optical thicknesses are finite although much larger for horizon than for radial paths.

The normal optical thickness of an atmosphere in which the number density of scatterers decreases exponentially with height z above the surface, $\exp(-z/H)$, is the same as that for a uniform atmosphere of finite thickness:

$$\tau_n = \int_0^\infty \beta \, dz = \beta_0 H, \tag{8}$$

where H is the *scale height* and β_0 is the scattering coefficient at sea level. This equivalence yields a good approximation even for the tangential optical thickness. For any zenith angle, the optical thickness is given approximately by

$$\frac{\tau}{\tau_n} = \sqrt{\frac{R_e^2}{H^2}\cos^2\Theta + \frac{2R_e}{H} + 1} - \frac{R_e}{H}\cos\Theta, \tag{9}$$

where R_e is the radius of the Earth. A flat earth is one for which R_e is infinite, in which instance Eq. (9) yields the expected relation

$$\lim_{R_e \to \infty} \frac{\tau}{\tau_n} = \frac{1}{\cos \Theta}. \qquad (10)$$

For Earth's atmosphere, the molecular scale height is about 8 km. According to the approximate relation Eq. (9), therefore, the horizon optical thickness is about 39 times greater than the normal optical thickness. Taking the exponential decrease of molecular number density into account yields a value about 10% lower.

Variations on the theme of reds and oranges at sunrise and sunset can be seen even when the sun is overhead. The radiance at an observer an optical distance τ from a (horizon) cloud is the sum of cloudlight transmitted to the observer and airlight:

$$L = L_0 G(1 - e^{-\tau}) + L_0 G_c e^{-\tau}, \qquad (11)$$

where G_c is a geometrical factor that accounts for scattering of nearly monodirectional sunlight into a hemisphere of directions by the cloud. If the cloud is approximated as an isotropic reflector with reflectance R and illuminated at an angle Φ, the geometrical factor G_c is $\Omega_s R \cos\Phi/\pi$, where Ω_s is the solid angle subtended by the sun at the Earth. If $G_c > G$, the observed radiance is redder (i.e., enriched in light of longer wavelengths) than the incident radiance. If $G_c < G$, the observed radiance is bluer than the incident radiance. Thus, distant horizon clouds can be reddish if they are bright or bluish if they are dark.

Underlying Eq. (11) is the implicit assumption that the line of sight is uniformly illuminated by sunlight. The first term in this equation is airlight; the second is transmitted cloudlight. Suppose, however, that the line of sight is shadowed from direct sunlight by clouds (that do not, of course, occlude the distant cloud of interest). This may reduce the first term in Eq. (11) so that the second term dominates. Thus, under a partly overcast sky, distant horizon clouds may be reddish even when the sun is high in the sky.

The zenith sky at sunset and twilight is the exception to the general rule that molecular scattering is sufficient to account for the color of the sky. In the absence of molecular absorption, the spectrum of the zenith sky would be essentially that of the zenith sun (although greatly reduced in radiance), hence would not be the blue that is observed. This was pointed out by Hulburt [7], who showed that absorption by ozone profoundly affects the color of the zenith sky when the sun is near the horizon. The Chappuis band of ozone extends from about 450 to 700 nm and peaks at around 600 nm. Preferential absorption of sunlight by ozone over long horizon paths gives the zenith sky its blueness when the sun is near the horizon. With the sun more than about 10° above the horizon, however, ozone has little effect on the color of the sky.

3
Polarization of Light in a Molecular Atmosphere

3.1
The Nature of Polarized Light

Unlike sound, light is a vector wave, an electromagnetic field lying in a plane normal to the propagation direction. The polarization state of such a wave is determined by the degree of correlation of any two orthogonal components into which its electric (or magnetic) field is resolved. Completely polarized light corresponds to complete correlation; completely unpolarized light corresponds to no correlation; partially polarized light corresponds to partial correlation.

If an electromagnetic wave is completely polarized, the tip of its oscillating electric field traces out a definite elliptical curve, the *vibration ellipse*. Lines and circles are special ellipses, the light being said to be linearly or circularly polarized, respectively. The general state of polarization is elliptical.

Any beam of light can be considered an incoherent superposition of two collinear beams, one unpolarized, the other completely polarized. The radiance of the polarized component relative to the total is defined as the *degree of polarization* (often multiplied by 100 and expressed as a percentage). This can be measured for a source of light (e.g., light from different sky directions) by rotating a (linear) polarizing filter and noting the minimum and maximum radiances transmitted by it. The degree of (linear) polarization is defined as the difference between these two radiances divided by their sum.

3.2
Polarization by Molecular Scattering

Unpolarized light can be transformed into partially polarized light upon interaction with matter because of different changes in amplitude of the two orthogonal field components. An example of this is the partial polarization of sunlight upon scattering by atmospheric molecules, which can be detected by looking at the sky through a polarizing filter (e.g., polarizing sunglasses) while rotating it. Waxing and waning of the observed brightness indicates some degree of partial polarization.

In the analysis of any scattering problem, a plane of reference is required. This is usually the *scattering plane*, determined by the directions of the incident and scattered waves, the angle between them being the *scattering angle*. Light polarized perpendicular (parallel) to the scattering plane is sometimes said to be vertically (horizontally) polarized. Vertical and horizontal in this context, however, are arbitrary terms indicating orthogonality and bear no relation, except by accident, to the direction of gravity.

The degree of polarization P of light scattered by a tiny sphere illuminated by unpolarized light is (Fig. 6)

$$P = \frac{1 - \cos^2\theta}{1 + \cos^2\theta}, \qquad (12)$$

where the scattering angle θ ranges from 0° (forward direction) to 180° (backward direction); the scattered light is partially linearly polarized perpendicular to the scattering plane. Although this equation

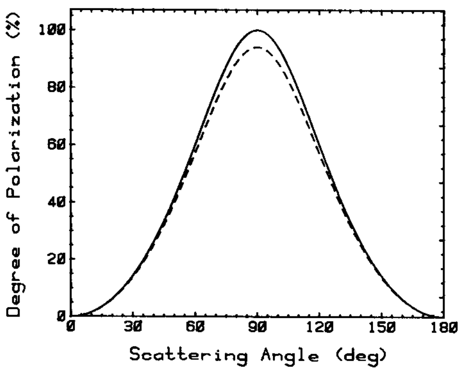

Fig. 6 Degree of polarization of the light scattered by a small (compared with the wavelength) sphere for incident unpolarized light (solid curve). The dashed curve is for a small spheroid chosen such that the degree of polarization at 90° is that for air

is a first step toward understanding polarization of skylight, more often than not it also has been a false step, having led countless authors to assert that skylight is completely polarized at 90° from the sun. Although $P = 1$ at $\theta = 90°$ according to Eq. (12), skylight is never 100% polarized at this or any other angle, and for several reasons.

Although air molecules are very small compared with the wavelengths of visible light, a requirement underlying Eq. (12), the dominant constituents of air are not spherically symmetric.

The simplest model of an asymmetric molecule is a small spheroid. Although it is indeed possible to find a direction in which the light scattered by such a spheroid is 100% polarized, this direction depends on the spheroid's orientation. In an ensemble of randomly oriented spheroids, each contributes its mite to the total radiance in a given direction, but each contribution is partially polarized to varying degrees between 0 and 100%. It is impossible for beams of light to be incoherently superposed in such a way that the degree of polarization of the resultant is greater than the degree of polarization of the most highly polarized beam.

Because air is an ensemble of randomly oriented asymmetric molecules, sunlight scattered by air never is 100% polarized. The intrinsic departure from perfection is about 6%. Figure 6 also includes a curve for light scattered by randomly oriented spheroids chosen to yield 94% polarization at 90°. This angle is so often singled out that it may deflect attention from nearby scattering angles. Yet, the degree of polarization is greater than 50% for a range of scattering angles 70° wide centered about 90°.

Equation (12) applies to air, not to the atmosphere, the distinction being that in the atmosphere, as opposed to the laboratory, multiple scattering is not negligible. Also, atmospheric air is almost never free of particles and is illuminated by light reflected by the ground. We must take the atmosphere as it is, whereas in the laboratory we often can eliminate everything we consider extraneous.

Because of both multiple scattering and ground reflection, light from any direction in the sky is not, in general, made up solely of light scattered in a single direction relative to the incident sunlight but is a superposition of beams with different scattering histories, hence different degrees of polarization. As a consequence, even if air molecules were perfect spheres and the atmosphere were completely free of particles, skylight would not be 100% polarized at 90° to the sun or at any other angle.

Reduction of the maximum degree of polarization is not the only consequence of multiple scattering. According to Fig. 6, there should be two *neutral points* in the sky, directions in which skylight is unpolarized: directly toward and away from the sun. Because of multiple scattering, however, there are three such points. When the sun is higher than about 20° above the horizon there are neutral points within 20° of the sun, the *Babinet point* above it, the *Brewster point* below. They coincide when the sun is directly overhead and move apart as the sun descends. When the sun is lower than 20°, the *Arago point* is about 20° above the antisolar point, the direction opposite the sun.

One consequence of the partial polarization of skylight is that the colors of distant objects may change when viewed through a rotated polarizing filter. If the sun is high in the sky, horizontal airlight will have a fairly high degree of polarization. According to the previous section, airlight is bluish. But if it also is partially polarized, its radiance can be diminished with a polarizing filter. Transmitted cloudlight, however, is unpolarized. Because the radiance of airlight can be reduced more than that of cloudlight, distant clouds may change from white to yellow to orange when viewed through a rotated polarizing filter.

4
Scattering by Particles

Up to this point we have considered only an atmosphere free of particles, an idealized state rarely achieved in nature. Particles still would inhabit the atmosphere even if the human race were to vanish from the Earth. They are not simply by-products of the ''dark satanic mills'' of civilization.

All molecules of the same substance are essentially identical. This is not true of particles: They vary in shape and size, and may be composed of one or more homogeneous regions.

4.1
The Salient Differences between Particles and Molecules: Magnitude of Scattering

The distinction between scattering by molecules when widely separated and when packed together into a droplet is that between scattering by incoherent and coherent arrays. Isolated molecules are excited primarily by incident (external) light, whereas the same molecules forming a droplet are excited by incident light and by each other's scattered fields. The total power scattered by an incoherent array of molecules is the sum of their scattered powers. The total power scattered by a coherent array is the square of the total scattered field, which in turn is the sum of all the fields scattered by the individual molecules. For an incoherent array we *may* ignore the wave nature of light, whereas for a coherent array we *must* take it into account.

Water vapor is a good example to ponder because it is a constituent of air and can condense to form cloud droplets. The difference between a sky containing water vapor and the same sky with the same amount of water but in the form of a cloud of droplets is dramatic.

According to Rayleigh's law, scattering by a particle small compared with the wavelength increases as the sixth power of its size (volume squared). A droplet of diameter 0.03 μm, for example, scatters about 10^{12} times more light than does one of its constituent molecules. Such a droplet contains about 10^7 molecules. Thus, scattering per molecule as a consequence of condensation of water vapor into a coherent water droplet increases by about 10^5.

Cloud droplets are much larger than 0.03 μm, a typical diameter being about 10 μm. Scattering per molecule in such a droplet is much greater than scattering by an isolated molecule, but not to the extent given by Rayleigh's law. Scattering increases as the sixth power of droplet diameter only when the molecules scatter coherently in phase. If a droplet is sufficiently small compared with the wavelength, each of its molecules is excited by essentially the same field and all the waves scattered by them interfere constructively. But when a droplet is comparable to or larger than the wavelength, interference can be constructive, destructive, and everything in between, and hence scattering does not increase as rapidly with droplet size as predicted by Rayleigh's law.

The figure of merit for comparing scatterers of different size is their scattering cross section per unit volume, which, except for a multiplicative factor, is the scattering cross section per molecule. A scattering cross section may be looked upon as an effective area for removing radiant energy from a beam: the scattering cross section times the beam irradiance is the radiant power scattered in all directions.

The scattering cross section per unit volume for water droplets illuminated by visible light and varying in size from molecules (10^{-4} μm) to raindrops (10^3 μm) is shown in Fig. 7. Scattering by a molecule that belongs to a cloud droplet is about 10^9 times greater than scattering by an isolated molecule, a striking example of the virtue of cooperation. Yet in molecular as in human societies there are limits beyond which cooperation becomes dysfunctional: Scattering by a molecule that belongs to a raindrop is about 100 times less than scattering by a molecule that belongs to a cloud droplet. This tremendous variation of scattering by water molecules depending on their state of aggregation has profound observational consequences. A cloud is optically so much different from the water vapor out of which it was born that the offspring bears no resemblance to its parents. We can see through tens of kilometers of air laden with water vapor, whereas a cloud a few tens of meters thick is enough to occult the sun. Yet a rainshaft born out of a cloud is considerably more translucent than its parent.

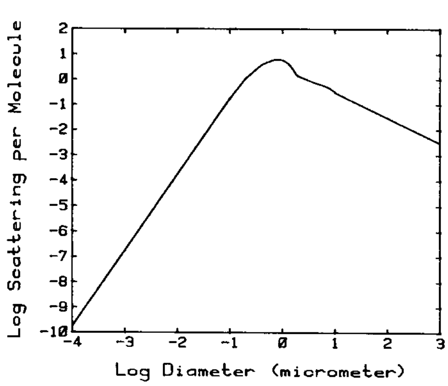

Fig. 7 Scattering (per molecule) of visible light (arbitrary units) by water droplets varying in size from a single molecule to a raindrop

4.2 The Salient Differences between Particles and Molecules: Wavelength Dependence of Scattering

Regardless of their size and composition, particles scatter approximately as the inverse fourth power of wavelength if they are small compared with the wavelength and absorption is negligible, two important caveats. Failure to recognize them has led to errors, such as that yellow light penetrates fog better because it is not scattered as much as light of shorter wavelengths. Although there may be perfectly sound reasons for choosing yellow instead of blue or green as the color of fog lights, greater transmission through fog is not one of them: Scattering by fog droplets is essentially independent of wavelength over the visible spectrum.

Small particles are selective scatterers; large particles are not. Particles neither small nor large give the reverse of what we have come to expect as normal. Figure 8 shows scattering of visible light by oil droplets with diameters 0.1, 0.8, and 10 μm. The smaller droplets scatter according to Rayleigh's law; the larger droplets (typical cloud droplet size) are nonselective. Between these two extremes are droplets (0.8 μm) that scatter long-wavelength light more than short-wavelength. Sunlight or moonlight seen

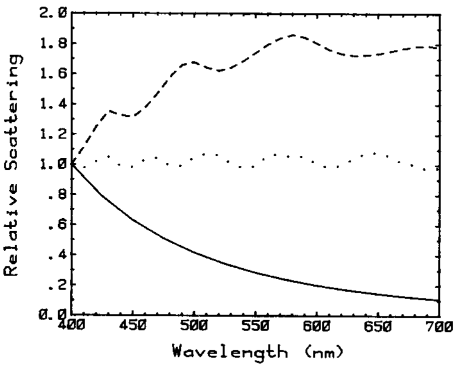

Fig. 8 Scattering of visible light by oil droplets of diameter 0.1 μm (solid curve), 0.8 μm (dashes), and 10 μm (dots)

through a thin cloud of these intermediate droplets would be bluish or greenish. This requires droplets of just the right size, and hence it is a rare event, so rare that it occurs once in a blue moon. Astronomers, for unfathomable reasons, refer to the second full moon in a month as a blue moon, but if such a moon were blue it would be only by coincidence. The last reliably reported outbreak of blue and green suns and moons occurred in 1950 and was attributed to an oily smoke produced in Canadian forest fires.

4.3
The Salient Differences between Particles and Molecules: Angular Dependence of Scattering

The angular distribution of scattered light changes dramatically with the size of the scatterer. Molecules and particles that are small compared with the wavelength are nearly isotropic scatterers of unpolarized light, the ratio of maximum (at 0° and 180°) to minimum (at 90°) scattered radiance being only 2 for spheres, and slightly less for other spheroids. Although small particles scatter the same in the forward and backward hemispheres, scattering becomes markedly asymmetric for particles comparable to or larger than the wavelength. For example, forward scattering by a water droplet as small as 0.5 μm is about 100 times greater than backward scattering, and the ratio of forward to backward scattering increases more or less monotonically with size (Fig. 9).

The reason for this asymmetry is found in the singularity of the forward direction. In this direction, waves scattered by two or more scatterers excited solely by incident light (ignoring mutual excitation) are always in phase regardless of the wavelength and the separation of the scatterers. If we imagine a particle to be made up of N small subunits, scattering in the forward direction increases as N^2, the only direction for which this is always true. For other directions, the wavelets scattered by the subunits will not necessarily all be in phase. As a consequence, scattering in the forward direction increases with size (i.e., N) more rapidly than in any other direction.

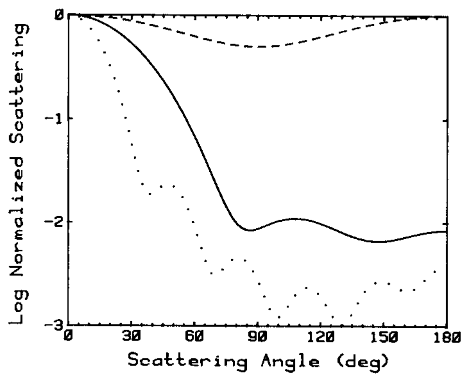

Fig. 9 Angular dependence of scattering of visible light (0.55 μm) by water droplets small compared with the wavelength (dashes), diameter 0.5 μm (solid curve), and diameter 10 μm (dots)

Many common observable phenomena depend on this forward-backward asymmetry. Viewed toward the illuminating sun, glistening fog droplets on a spider's web warn us of its presence. But when we view the web with our backs to the sun, the web mysteriously disappears. A pattern of dew illuminated by the rising sun on a cold morning seems etched on a windowpane. But if we go outside to look at the window, the pattern vanishes. Thin clouds sometimes hover over warm, moist heaps of dung, but may go unnoticed unless they lie between us and the source of illumination. These are but a few examples of the consequences of strongly asymmetric scattering by single particles comparable to or larger than the wavelength.

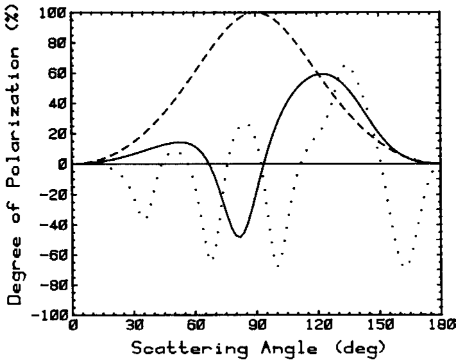

Fig. 10 Degree of polarization of light scattered by water droplets illuminated by unpolarized visible light (0.55 μm). The dashed curve is for a droplet small compared with the wavelength; the solid curve is for a droplet of diameter 0.5 μm; the dotted curve is for a droplet of diameter 1.0 μm. Negative degrees of polarization indicate that the scattered light is partially polarized parallel to the scattering plane

4.4
The Salient Differences between Particles and Molecules: Degree of Polarization of Scattered Light

All the simple rules about polarization upon scattering are broken when we turn from molecules and small particles to particles comparable to the wavelength. For example, the degree of polarization of light scattered by small particles is a simple function of scattering angle. But simplicity gives way to complexity as particles grow (Fig. 10), the scattered light being partially polarized parallel to the scattering plane for some scattering angles, perpendicular for others.

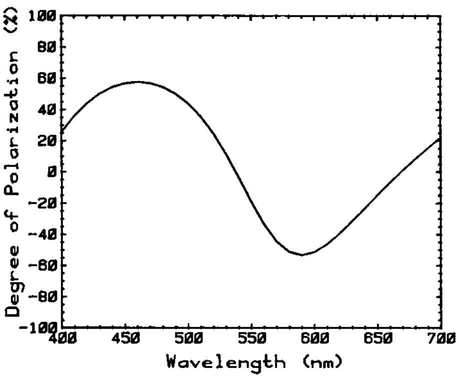

Fig. 11 Degree of polarization at a scattering angle of 90° of light scattered by a water droplet of diameter 0.5 μm illuminated by unpolarized light

The degree of polarization of light scattered by molecules or by small particles is essentially independent of wavelength. But this is not true for particles comparable to or larger than the wavelength. Scattering by such particles exhibits *dispersion of polarization*: The degree of polarization at, say, 90° may vary considerably over the visible spectrum (Fig. 11).

In general, particles can act as polarizers or retarders or both. A polarizer transforms unpolarized light into partially polarized light. A retarder transforms polarized light of one form into that of another (e.g., linear into elliptical). Molecules and small particles, however, are restricted to roles as polarizers. If the atmosphere were inhabited solely by such scatterers, skylight could never be other than partially linearly polarized. Yet particles comparable to or larger than the wavelength often are present; hence skylight can acquire a degree of ellipticity upon multiple scattering: Incident unpolarized light is partially linearly polarized in the first scattering event, then transformed into partially elliptically polarized light in subsequent events.

Bees can navigate by polarized skylight. This statement, intended to evoke great awe for the photopolimetric powers of bees, is rarely accompanied by an important caveat: The sky must be clear. Figures 10 and 11 show two reasons – there are others – why bees, remarkable though they may be, cannot do the impossible. The simple wavelength-independent relation between the position of the sun and the direction in which skylight is most highly polarized, an underlying necessity for navigating by means of polarized skylight, is obliterated when clouds cover the sky. This was recognized by the decoder of bee dances himself von Frisch, [8]: "Sometimes a cloud would pass across the area of sky visible through the tube; when this happened the dances became disoriented, and the bees were unable to indicate the direction to the feeding place. Whatever phenomenon in the blue sky served to orient the dances, this experiment showed that it was seriously disturbed if the blue sky was covered by a cloud." But von Frisch's words often have been forgotten by disciples eager to spread the story about bee magic to those just as eager to believe what is charming even though untrue.

4.5
The Salient Differences between Particles and Molecules: Vertical Distributions

Not only are the scattering properties of particles quite different, in general, from those of molecules; the different vertical distributions of particles and molecules by themselves affect what is observed. The number density of molecules decreases more or less exponentially with height z above the surface: $\exp(-z/H_m)$, where the molecular scale height H_m is around 8 km. Although the decrease in number density of particles with height is also approximately exponential, the scale height for particles H_p is about 1–2 km. As a consequence, particles contribute disproportionately to optical thicknesses along near-horizon paths. Subject to the approximations underlying Eq. (9), the ratio of the tangential (horizon) optical thickness for particles τ_{tp} to that for molecules τ_{tm} is

$$\frac{\tau_{tp}}{\tau_{tm}} = \frac{\tau_{np}}{\tau_{nm}}\sqrt{\frac{H_m}{H_p}}, \qquad (13)$$

where the subscript t indicates a tangential path and n indicates a normal (radial) path. Because of the incoherence of scattering by atmospheric molecules and particles, scattering coefficients are additive, and hence so are optical thicknesses. For equal normal optical thicknesses, the tangential optical thickness for particles is at least twice that for molecules. Molecules by themselves cannot give red sunrises and sunsets; molecules need the help of particles. For a fixed τ_{np}, the tangential optical thickness for particles is greater the more they are concentrated near the ground.

At the horizon the relative rate of change of transmission T of sunlight with zenith angle is

$$\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}\Theta} = \tau_n \frac{R_e}{H}, \tag{14}$$

where the scale height and normal optical thickness may be those for molecules or particles. Not only do particles, being more concentrated near the surface, give disproportionate attenuation of sunlight on the horizon, but they magnify the angular gradient of attenuation there. A perceptible change in color across the sun's disk (which subtends about 0.5°) on the horizon also requires the help of particles.

Fig. 12 Because of scattering by molecules and particles along the line of sight, each successive ridge is brighter than the ones in front of it even though all of them are covered with the same dark vegetation

5 Atmospheric Visibility

On a clear day can we really see forever? If not, how far can we see? To answer this question requires qualifying it by restricting viewing to more or less horizontal paths during daylight. Stars at staggering distances can be seen at night, partly because there is no skylight to reduce contrast, partly because stars overhead are seen in directions for which attenuation by the atmosphere is least.

The radiance in the direction of a black object is not zero, because of light scattered along the line of sight (see Sec. 2.4). At sufficiently large distances, this airlight is indistinguishable from the horizon sky. An example is a phalanx of parallel dark ridges, each ridge less distinct than those in front of it (Fig. 12). The farthest ridges blend into the horizon sky. Beyond some distance we cannot see ridges because of insufficient contrast.

Equation (5) gives the airlight radiance, a radiometric quantity that describes radiant power without taking into account the portion of it that stimulates the human eye or by what relative amount it does so at each wavelength. Luminance (also sometimes called *brightness*) is the corresponding photometric quantity. Luminance and radiance are related by an integral over the visible spectrum:

$$B = \int K(\lambda)L(\lambda)\,\mathrm{d}\lambda, \tag{15}$$

where the luminous efficiency of the human eye K peaks at about 550 nm and vanishes outside the range 385–760 nm.

The *contrast* C between any object and the horizon sky is

$$C = \frac{B - B_\infty}{B_\infty}, \tag{16}$$

where B_∞ is the luminance for an infinite horizon optical thickness. For a uniformly illuminated line of sight of length d, uniform in its scattering properties, and with a black backdrop, the contrast is

$$C = -\frac{\displaystyle\int KGL_0 \exp(-\beta d)\,\mathrm{d}\lambda}{\displaystyle\int KGL_0\,\mathrm{d}\lambda}. \qquad (17)$$

The ratio of integrals in this equation defines an average optical thickness:

$$C = -\exp(-\langle\tau\rangle). \qquad (18)$$

This expression for contrast reduction with (optical) distance is mathematically, but not physically, identical to Eq. (6), which perhaps has engendered the misconception that atmospheric visibility is reduced because of attenuation. Yet as there is no light from a black object to be attenuated, its finite visual range cannot be a consequence of attenuation.

The distance beyond which a dark object cannot be distinguished from the horizon sky is determined by the *contrast threshold*: the smallest contrast detectable by the human observer. Although this depends on the particular observer, the angular size of the object observed, the presence of nearby objects, and the absolute luminance, a contrast threshold of 0.02 is often taken as an average. This value in Eq. (18) gives

$$-\ln|C| = 3.9 = \langle\tau\rangle = \langle\beta d\rangle. \qquad (19)$$

To convert an optical distance into a physical distance requires the scattering coefficient. Because K is peaked at around 550 nm, we can obtain an approximate value of d from the scattering coefficient at this wavelength in Eq. (19). At sea level, the molecular scattering coefficient in the middle of the visible spectrum corresponds to about 330 km for "forever": the greatest distance at which a black object can be seen against the horizon sky assuming a contrast threshold of 0.02 and ignoring the curvature of the earth.

We also observe contrast between elements of the same scene, a hillside mottled with stands of trees and forest clearings, for example. The extent to which we can resolve details in such a scene depends on sun angle as well as distance.

The airlight radiance for a nonreflecting object is Eq. (5) with $G = p(\Theta)\Omega_s$, where $p(\Theta)$ is the probability (per unit solid angle) that light is scattered in a direction making an angle Θ with the incident sunlight and Ω_s is the solid angle subtended by the sun. When the sun is overhead, $\Theta = 90°$; with the sun at the observer's back, $\Theta = 180°$; for an observer looking directly into the sun, $\Theta = 0°$.

The radiance of an object with a finite reflectance R and illuminated at an angle Φ is given by Eq. (11). Equations (5) and (11) can be combined to obtain the contrast between reflecting and nonreflecting objects:

$$C = \frac{F\mathrm{e}^{-\tau}}{1 + (F-1)\mathrm{e}^{-\tau}}, \qquad F = \frac{R\cos\Phi}{\pi p(\Theta)}. \qquad (20)$$

All else being equal, therefore, contrast decreases as $p(\Theta)$ increases. As shown in Fig. 9, $p(\Theta)$ is more sharply peaked in the forward direction the larger the scatterer. Thus, we expect the details of a distant scene to be less distinct when looking toward the sun than away from it if the optical thickness of the line of sight has an appreciable component contributed by particles comparable to or larger than the wavelength.

On humid, hazy days, visibility is often depressingly poor. Haze, however, is not water vapor but rather water that has ceased to be vapor. At high relative humidities, but still well below

100%, small soluble particles in the atmosphere accrete liquid water to become solution droplets (haze). Although these droplets are much smaller than cloud droplets, they markedly diminish visual range because of the sharp increase in scattering with particle size (Fig. 7). The same number of water molecules when aggregated in haze scatter vastly more than when apart.

6 Atmospheric Refraction

6.1 Physical Origins of Refraction

Atmospheric refraction is a consequence of molecular scattering, which is rarely stated given the historical accident that before light and matter were well understood refraction and scattering were locked in separate compartments and subsequently have been sequestered more rigidly than monks and nuns in neighboring cloisters.

Consider a beam of light propagating in an optically homogeneous medium. Light is scattered (weakly but observably) laterally to this beam as well as in the direction of the beam (the forward direction). The observed beam is a coherent superposition of incident light and forward-scattered light, which was excited by the incident light. Although refractive indices are often defined by ratios of phase velocities, we may also look upon a refractive index as a parameter that specifies the phase shift between an incident beam and the forward-scattered beam that the incident beam excites. The connection between (incoherent) scattering and refraction (coherent scattering) can be divined from the expressions for the refractive index n of a gas and the scattering cross section σ_s of a gas molecule:

$$n = 1 + \tfrac{1}{2}\alpha N, \tag{21}$$

$$\sigma_s = \frac{k^4}{6\pi}|\alpha|^2, \tag{22}$$

where N is the number density (not mass density) of gas molecules, $k = 2\pi/\lambda$ is the wave number of the incident light, and α is the polarizability of a molecule (induced dipole moment per unit inducing electric field). The appearance of the polarizability in Eq. (21) but its square in Eq. (22) is the clue that refraction is associated with electric fields whereas lateral scattering is associated with electric fields squared (powers). Scattering, without qualification, often means incoherent scattering in all directions. Refraction, in a nutshell, is coherent scattering in a particular direction.

Readers whose appetites have been whetted by the preceding brief discussion of the physical origins of refraction are directed to a beautiful paper by Doyle [9] in which he shows how the Fresnel equations can be dissected to reveal the scattering origins of (specular) reflection and refraction.

6.2 Terrestrial Mirages

Mirages are not illusions, any more so than are reflections in a pond. Reflections of plants growing at its edge are not interpreted as plants growing into the water. If the water is ruffled by wind, the reflected images may be so distorted that they are no longer recognizable as those of plants. Yet we still would not call such distorted images illusions. And so is it with mirages. They are images noticeably different from what they would be in the absence of atmospheric refraction, creations of the atmosphere, not of the mind.

Mirages are vastly more common than is realized. Look and you shall see them. Contrary to popular opinion, they are not unique to deserts. Mirages can be seen frequently even over ice-covered landscapes and highways flanked by deep snowbanks. Temperature *per se* is not what gives mirages but rather temperature gradients.

Because air is a mixture of gases, the polarizability for air in Eq. (21) is an average over all its molecular constituents, although their individual polarizabilities are about the same (at visible wavelengths). The vertical refractive index gradient can be written so as to show its dependence on pressure p and (absolute) temperature T:

$$\frac{d}{dz}\ln(n-1) = \frac{1}{p}\frac{dp}{dz} - \frac{1}{T}\frac{dT}{dz}. \qquad (23)$$

Pressure decreases approximately exponentially with height, where the scale height is around 8 km. Thus, the first term on the right-hand side of Eq. (23) is around 0.1 km^{-1}. Temperature usually decreases with height in the atmosphere. An average lapse rate of temperature (i.e., its decrease with height) is around 6 °C/km. The average temperature in the troposphere (within about 15 km of the surface) is around 280 K. Thus, the magnitude of the second term in Eq. (23) is around 0.02 km^{-1}. On average, therefore, the refractive-index gradient is dominated by the vertical pressure gradient. But within a few meters of the surface, conditions are far from average. On a sun-baked highway your feet may be touching asphalt at 50 °C while your nose is breathing air at 35 °C, which corresponds to a lapse rate a thousand times the average. Moreover, near the surface, temperature can increase with height. In shallow surface layers, in which the pressure is nearly constant, the temperature gradient determines the refractive index gradient. It is in such shallow layers that mirages, which are caused by refractive-index gradients, are seen.

Cartoonists by their fertile imaginations unfettered by science, and textbook writers by their carelessness, have engendered the notion that atmospheric refraction can work wonders, lifting images of ships, for example, from the sea high into the sky. A back-of-the-envelope calculation dispels such notions. The refractive index of air at sea level is about 1.0003 (Fig. 13). Light from empty space incident at glancing incidence onto a uniform slab with this refractive index is displaced in angular position from where it would have been in the absence of refraction by

$$\delta = \sqrt{2(n-1)}. \qquad (24)$$

This yields an angular displacement of about 1.4°, which as we shall see is a rough upper limit.

Trajectories of light rays in nonuniform media can be expressed in different ways. According to Fermat's principle of least

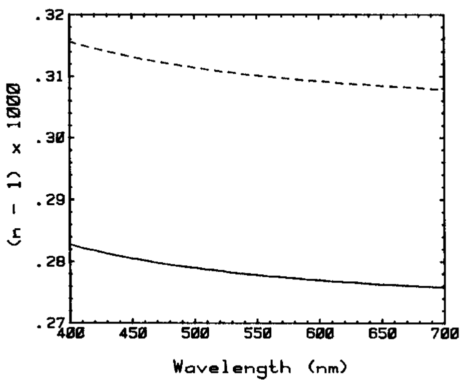

Fig. 13 Sea-level refractive index versus wavelength at −15 °C (dashes) and 15 °C (solid curve). Data from Penndorf, R. (**1957**), *J. Opt. Soc. Am.* **47**, 176–182 [2]

time (which ought to be extreme time), the actual path taken by a ray between two points is such that the path integral

$$\int_1^2 n\,\mathrm{d}s \qquad (25)$$

is an extremum over all possible paths. This principle has inspired piffle about the alleged efficiency of nature, which directs light over routes that minimize travel time, presumably freeing it to tend to important business at its destination.

The scale of mirages is such that in analyzing them we may pretend that the Earth is flat. On such an earth, with an atmosphere in which the refractive index varies only in the vertical, Fermat's principle yields a generalization

$$n \sin\theta = \text{constant} \qquad (26)$$

of Snel's law, where θ is the angle between the ray and the vertical direction. We could, of course, have bypassed Fermat's principle to obtain this result.

If we restrict ourselves to nearly horizontal rays, Eq. (26) yields the following differential equation satisfied by a ray:

$$\frac{\mathrm{d}^2 z}{\mathrm{d}y^2} = \frac{\mathrm{d}n}{\mathrm{d}z}, \qquad (27)$$

where y and z are its horizontal and vertical coordinates, respectively. For a constant refractive-index gradient, which to good approximation occurs for a constant temperature gradient, Eq. (27) yields parabolas for ray trajectories. One such parabola for a constant temperature gradient about 100 times the average is shown in Fig. 14. Note the vastly different horizontal and vertical scales. The image is displaced downward from what it would be in the absence of atmospheric refraction; hence the designation *inferior* mirage. This is the familiar highway mirage, seen over highways warmer than the air above them. The downward angular displacement is

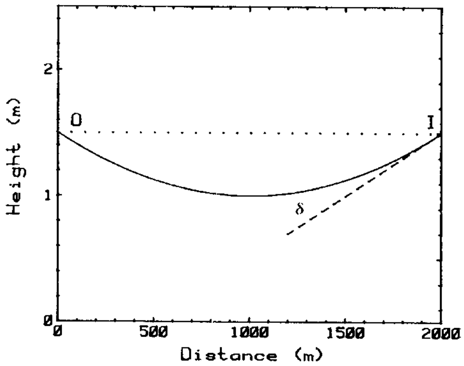

Fig. 14 Parabolic ray paths in an atmosphere with a constant refractive-index gradient (inferior mirage). Note the vastly different horizontal and vertical scales

$$\delta = \frac{1}{2} s \frac{\mathrm{d}n}{\mathrm{d}z}, \qquad (28)$$

where s is the horizontal distance between object and observer (image). Even for a temperature gradient 1000 times the tropospheric average, displacements of mirages are less than a degree at distances of a few kilometers.

If temperature increases with height, as it does, for example, in air over a cold sea, the resulting mirage is called a *superior mirage.* Inferior and superior are not designations of lower and higher caste but rather of displacements downward and upward.

For a constant temperature gradient, one and only one parabolic ray trajectory connects an object point to an image point. Multiple images therefore are not possible. But temperature gradients close to the ground are rarely linear. The upward transport of energy from a hot surface occurs by molecular conduction through a stagnant boundary layer of air. Somewhat above the surface, however, energy is transported by air in motion. As a consequence, the temperature gradient steepens toward the ground if the energy flux is constant. This variable gradient can lead to two observable consequences: magnification and multiple images.

According to Eq. (28), all image points at a given horizontal distance are displaced downward by an amount proportional to the (constant) refractive index gradient. A corollary is that the closer an object point is to a surface, where the temperature gradient is greatest, the greater the downward displacement of the corresponding image point. Thus, nonlinear vertical temperature profiles may magnify images.

Multiple images are seen frequently on highways. What often appears to be water on the highway ahead but evaporates before it is reached is the inverted secondary image of either the horizon sky or horizon objects lighter than dark asphalt.

6.3 Extraterrestrial Mirages

When we turn from mirages of terrestrial objects to those of extraterrestrial bodies, most notably the sun and moon, we can no longer pretend that the Earth is flat. But we can pretend that the atmosphere is uniform and bounded. The total phase shift of a vertical ray from the surface to infinity is the same in an atmosphere with an exponentially decreasing molecular number density as in a hypothetical atmosphere with a uniform number density equal to the surface value up to height H.

A ray refracted along a horizon path by this hypothetical atmosphere and originating from outside it had to have been incident on it from an angle δ below the horizon:

$$\delta = \sqrt{\frac{2H}{R}} - \sqrt{\frac{2H}{R} - 2(n-1)}, \qquad (29)$$

where R is the radius of the Earth. Thus, when the sun (or moon) is seen to be on the horizon it is actually more than halfway below it, δ being about 0.36°, whereas the angular width of the sun (or moon) is about 0.5°.

Extraterrestrial bodies seen near the horizon also are vertically compressed. The simplest way to estimate the amount of compression is from the rate of change of angle of refraction θ_r with angle of incidence θ_i for a uniform slab

$$\frac{d\theta_r}{d\theta_i} = \frac{\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}, \qquad (30)$$

where the angle of incidence is that for a curved but uniform atmosphere such that the refracted ray is horizontal. The result is

$$\frac{d\theta_r}{d\theta_i} = \sqrt{1 - \frac{R}{H}(n-1)}, \qquad (31)$$

according to which the sun near the horizon is distorted into an ellipse with aspect ratio about 0.87. We are unlikely to notice this distortion, however, because we expect the sun and moon to be circular, and hence we see them that way.

The previous conclusions about the downward displacement and distortion of the sun were based on a refractive-index profile determined mostly by the vertical pressure gradient. Near the ground, however, the temperature gradient is the prime determinant of the refractive-index gradient, as a consequence of which the sun on the horizon can take on shapes

Fig. 15 A nearly triangular sun on the horizon. The serrations are a consequence of horizontal variations in refractive index

more striking than a mere ellipse. For example, Fig. 15 shows a nearly triangular sun with serrated edges. Assigning a cause to these serrations provides a lesson in the perils of jumping to conclusions. Obviously, the serrations are the result of sharp changes in the temperature gradient – or so one might think. Setting aside how such changes could be produced and maintained in a real atmosphere, a theorem of Fraser [10] gives pause for thought. According to this theorem, "In a horizontally (spherically) homogeneous atmosphere it is impossible for more than one image of an extraterrestrial object (sun) to be seen above the astronomical horizon." The serrations on the sun in Fig. 15 are multiple images. But if the refractive index varies only vertically (i.e., along a radius), no matter how sharply, multiple images are not possible. Thus, the serrations must owe their existence to horizontal variations of the refractive index, a consequence of gravity waves propagating along a temperature inversion.

6.4 The Green Flash

Compared to the rainbow, the green flash is not a rare phenomenon. Before you dismiss this assertion as the ravings of a lunatic, consider that rainbows require raindrops as well as sunlight to illuminate them, whereas rainclouds often completely obscure the sun. Moreover, the sun must be below about 42°. As a consequence of these conditions, rainbows are not seen often, but often enough that they are taken as the paragon of color variation. Yet tinges of green on the upper rim of the sun can be seen every day at sunrise and sunset given a sufficiently low horizon and a cloudless sky. Thus, the conditions for seeing a green flash are more easily met than those for seeing a rainbow. Why then is the green flash considered to be so rare? The distinction here is that between a rarely observed phenomenon (the green flash) and a rarely observable one (the rainbow).

The sun may be considered to be a collection of disks, one for each visible wavelength. When the sun is overhead, all the disks coincide and we see the sun as white. But as it descends in the sky, atmospheric refraction displaces the disks by slightly different amounts, the red less than the violet (see Fig. 13). Most of each disk overlaps all the others except for the disks at the extremes of the visible spectrum. As a consequence, the upper rim of the sun is violet or blue, its lower rim red, whereas its interior, the region in which all disks overlap, is still white.

This is what would happen in the absence of lateral scattering of sunlight. But refraction and lateral scattering go hand in hand, even in an atmosphere free of particles. Selective scattering by atmospheric molecules and particles causes the color of the sun to change. In particular, the violet-bluish upper rim of the low sun can be transformed to green.

According to Eq. (29) and Fig. 13, the angular width of the green upper rim of the low sun is about 0.01°, too narrow to be resolved with the naked eye or even to be seen against its bright backdrop. But depending on the temperature profile, the atmosphere itself can magnify the upper rim and yield a second image of it, thereby enabling it to be seen without the aid of a telescope or binoculars. Green rims, which require artificial magnification, can be seen more frequently than green flashes, which require natural magnification. Yet both can be seen often by those who know what to look for and are willing to look.

7
Scattering by Single Water Droplets

All the colored atmospheric displays that result when water droplets (or ice crystals) are illuminated by sunlight have the same underlying cause: light is scattered in different amounts in different directions by particles larger than the wavelength, and the directions in which scattering is greatest depends on wavelength. Thus, when particles are illuminated by white light, the result can be angular separation of colors even if scattering integrated over all directions is independent of wavelength (as it essentially is for cloud droplets and ice crystals). This description, although correct, is too general to be completely satisfying. We need something more specific, more quantitative, which requires theories of scattering.

Because superficially different theories have been used to describe different optical phenomena, the notion has become widespread that they are caused by these theories. For example, coronas are said to be caused by diffraction and rainbows by refraction. Yet both the corona and the rainbow can be described quantitatively to high accuracy with a theory (the Mie theory for scattering by a sphere) in which diffraction and refraction do not explicitly appear. No fundamentally impenetrable barrier separates scattering from (specular) reflection, refraction, and diffraction. Because these terms came into general use and were entombed in textbooks before the nature of light and matter was well understood, we are stuck with them. But if we insist that diffraction, for example, is somehow different from scattering, we do so at the expense of shattering the unity of the seemingly disparate observable phenomena that result when light interacts with matter. What is observed depends on the composition and disposition of the matter, not on which approximate theory in a hierarchy is used for quantitative description.

Atmospheric optical phenomena are best classified by the direction in which they are seen and by the agents responsible for them. Accordingly, the following sections are arranged in order of scattering direction, from forward to backward.

When a single water droplet is illuminated by white light and the scattered light projected onto a screen, the result is a set of colored rings. But in the atmosphere we see a mosaic to which individual droplets contribute. The scattering pattern of a single droplet is the same as the mosaic provided that multiple scattering is negligible.

7.1
Coronas and Iridescent Clouds

A cloud of droplets narrowly distributed in size and thinly veiling the sun (or moon) can yield a spectacular series of colored concentric rings around it. This corona is most easily described quantitatively by the Fraunhofer diffraction theory, a simple approximation valid for particles large compared with the wavelength and for scattering angles near the forward direction. According to this approximation, the differential scattering cross section (cross section for scattering into a unit solid angle) of a spherical droplet of radius a illuminated by light of wave number k is

$$\frac{|S|^2}{k^2}, \qquad (32)$$

where the *scattering amplitude* is

$$S = x^2 \frac{1 + \cos\theta}{2} \frac{J_1(x \sin\theta)}{x \sin\theta}. \qquad (33)$$

The term J_1 is the Bessel function of first order and the size parameter $x = ka$. The quantity $(1 + \cos\theta)/2$ is usually approximated by 1 since only near-forward scattering angles θ are of interest.

The differential scattering cross section, which determines the angular distribution of the scattered light, has maxima for $x \sin\theta = 5.137,\ 8.417,\ 11.62,\ \ldots$ Thus, the dispersion in the position of the first maximum is

$$\frac{d\theta}{d\lambda} \approx \frac{0.817}{a} \qquad (34)$$

and is greater for higher-order maxima. This dispersion determines the upper limit on drop size such that a corona can be observed. For the total angular dispersion over the visible spectrum to be greater than the angular width of the sun (0.5°), the droplets cannot be larger than about 60 μm in diameter. Drops in rain, even in drizzle, are appreciably larger than this, which is why coronas are not seen through rainshafts. Scattering by a droplet of diameter 10 μm (Fig. 16), a typical cloud droplet size, gives sufficient dispersion to yield colored coronas.

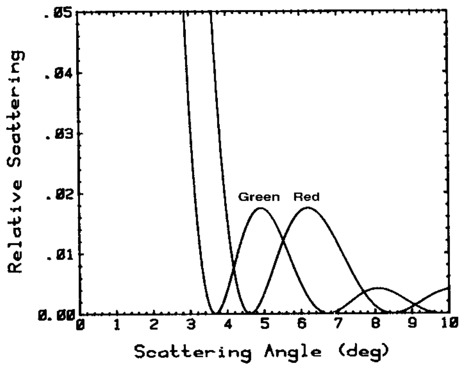

Fig. 16 Scattering of light near the forward direction (according to Fraunhofer theory) by a sphere of diameter 10 μm illuminated by red and green light

Suppose that the first angular maximum for blue light (0.47 μm) occurs for a droplet of radius a. For red light (0.66 μm) a maximum is obtained at the same angle for a droplet of radius $a + \Delta a$. That is, the two maxima, one for each wavelength, coincide. From this we conclude that coronas require narrow size distributions: if cloud droplets are distributed in radius with a relative variance $\Delta a/a$ greater than about 0.4, color separation is not possible.

Because of the stringent requirements for the occurrence of coronas, they are not observed often. Of greater occurrence are the corona's cousins, iridescent clouds, which display colors but usually not arranged in any obviously regular geometrical pattern. Iridescent patches in clouds can be seen even at the edges of thick clouds that occult the sun.

Coronas are not the unique signatures of spherical scatterers. Randomly oriented ice columns and plates give similar patterns according to Fraunhofer theory [11]. As a practical matter, however, most coronas probably are caused by droplets. Many clouds at temperatures well below freezing contain subcooled water droplets. Only if a corona were seen in a cloud at a temperature lower than −40 °C could one assert with confidence that it must be an ice-crystal corona.

7.2 Rainbows

In contrast with coronas, which are seen looking toward the sun, rainbows are seen looking away from it, and are caused by water drops much larger than those that give coronas. To treat the rainbow quantitatively we may pretend that light incident on a transparent sphere is composed of individual rays, each of which suffers a different fate determined only by the laws of specular reflection and refraction. Theoretical justification for this is provided by van de Hulst's ([12], p. 208) *localization principle*, according to which terms in the exact solution for scattering by a transparent sphere correspond to more or less localized rays.

Each incident ray splinters into an infinite number of scattered rays: externally reflected, transmitted without internal reflection, transmitted after one, two, and so on internal reflections. At any scattering angle θ, each splinter contributes to the scattered light. Accordingly, the differential scattering cross section is an infinite series with terms of the form

$$\frac{b(\theta)}{\sin\theta}\frac{db}{d\theta}. \qquad (35)$$

The *impact parameter* b is $a\sin\Theta_i$, where Θ_i is the angle between an incident ray and the normal to the sphere. Each term in the series corresponds to one of the splinters of an incident ray. A *rainbow angle* is a singularity (or *caustic*) of the differential scattering cross section at which the conditions

$$\frac{d\theta}{db} = 0, \quad \frac{b}{\sin\theta} \neq 0 \qquad (36)$$

are satisfied. Missing from Eq. (35) are various reflection and transmission coefficients (Fresnel coefficients), which display no singularities and hence do not determine rainbow angles.

A rainbow is not associated with rays externally reflected or transmitted without internal reflection. The succession of rainbow angles associated with one, two, three ... internal reflections are called *primary, secondary, tertiary* ... rainbows. Aristotle recognized that "Three or more rainbows are never seen, because even the second is dimmer than the first, and so the third reflection is altogether too feeble to reach the sun (Aristotle's view was that light streams outward from the eye)". Although he intuitively grasped that each successive ray is associated with ever-diminishing energy, his statement about the nonexistence of tertiary rainbows in nature is not quite true. Although reliable reports of such rainbows are rare (unreliable reports are as common as dirt), at least one observer who can be believed has seen one [13].

An incident ray undergoes a total angular deviation as a consequence of transmission into the drop, one or more internal reflections, and transmission out of the drop. Rainbow angles are angles of minimum deviation.

For a rainbow of any order to exist,

$$\cos\Theta_i = \sqrt{\frac{n^2-1}{p(p+1)}} \qquad (37)$$

must lie between 0 and 1, where Θ_i is the angle of incidence of a ray that gives a rainbow after p internal reflections and

n is the refractive index of the drop. A primary bow therefore requires drops with refractive index less than 2; a secondary bow requires drops with refractive index less than 3. If raindrops were composed of titanium dioxide ($n \approx 3$), a commonly used opacifier for paints, primary rainbows would be absent from the sky and we would have to be content with only secondary bows.

If we take the refractive index of water to be 1.33, the scattering angle for the primary rainbow is about 138°. This is measured from the forward direction (solar point). Measured from the antisolar point (the direction toward which one must look in order to see rainbows in nature), this scattering angle corresponds to 42°, the basis for a previous assertion that rainbows (strictly, primary rainbows) cannot be seen when the sun is above 42°. The secondary rainbow is seen at about 51° from the antisolar point. Between these two rainbows is *Alexander's dark band*, a region into which no light is scattered according to geometrical optics.

The colors of rainbows are a consequence of sufficient dispersion of the refractive index over the visible spectrum to give a spread of rainbow angles that appreciably exceeds the width of the sun. The width of the primary bow from violet to red is about 1.7°; that of the secondary bow is about 3.1°.

Because of its band of colors arcing across the sky, the rainbow has become the paragon of color, the standard against which all other colors are compared. Lee and Fraser [14, 15], however, challenged this status of the rainbow, pointing out that even the most vivid rainbows are colorimetrically far from pure.

Rainbows are almost invariably discussed as if they occurred literally in a vacuum. But real rainbows, as opposed to the pencil-and-paper variety, are necessarily observed in an atmosphere, the molecules and particles of which scatter sunlight that adds to the light from the rainbow but subtracts from its purity of color.

Although geometrical optics yields the positions, widths, and color separation of rainbows, it yields little else. For example, geometrical optics is blind to *supernumerary bows*, a series of narrow bands sometimes seen below the primary bow. These bows are a consequence of interference, and hence fall outside the province of geometrical optics. Since supernumerary bows are an interference phenomenon, they, unlike primary and secondary bows (according to geometrical optics), depend on drop size. This poses the question of how supernumerary bows can be seen in rain showers, the drops in which are widely distributed in size. In a nice piece of detective work, Fraser [16] answered this question.

Raindrops falling in a vacuum are spherical. Those falling in air are distorted by aerodynamic forces, not, despite the depictions of countless artists, into teardrops but rather into nearly oblate spheroids with their axes more or less vertical. Fraser argued that supernumerary bows are caused by drops with a diameter of about 0.5 mm, at which diameter the angular position of the first (and second) supernumerary bow has a minimum: interference causes the position of the supernumerary bow to increase with decreasing size whereas drop distortion causes it to increase with increasing size. Supernumerary patterns contributed by drops on either side of the minimum cancel, leaving only the contribution from drops at the minimum. This cancellation occurs only near the tops of rainbows, where supernumerary bows are seen. In the vertical parts of a rainbow, a horizontal slice through a distorted drop is more or less circular, and hence these drops do not exhibit a minimum supernumerary angle.

According to geometrical optics, all spherical drops, regardless of size, yield the same rainbow. But it is not necessary for a drop to be spherical for it to yield rainbows independent of its size. This merely requires that the plane defined by the incident and scattered rays intersect the drop in a circle. Even distorted drops satisfy this condition in the vertical part of a bow. As a consequence, the absence of supernumerary bows there is compensated for by more vivid colors of the primary and secondary bows [17]. Smaller drops are more likely to be spherical, but the smaller a drop, the less light it scatters. Thus, the dominant contribution to the luminance of rainbows is from the larger drops. At the top of a bow, the plane defined by the incident and scattered rays intersects the large, distorted drops in an ellipse, yielding a range of rainbow angles varying with the amount of distortion, and hence a pastel rainbow. To the knowledgeable observer, rainbows are no more uniform in color and brightness than is the sky.

Although geometrical optics predicts that all rainbows are equal (neglecting background light), real rainbows do not slavishly follow the dictates of this approximate theory. Rainbows in nature range from nearly colorless fog bows (or cloud bows) to the vividly colorful vertical portions of rainbows likely to have inspired myths about pots of gold.

7.3 The Glory

Continuing our sweep of scattering directions, from forward to backward, we arrive at the end of our journey: *the glory*. Because it is most easily seen from airplanes it sometimes is called the *pilot's bow*. Another name is *anticorona*, which signals that it is a corona around the antisolar point. Although glories and coronas share some common characteristics, there are differences between them other than direction of observation. Unlike coronas, which may be caused by nonspherical ice crystals, glories require spherical cloud droplets. And a greater number of colored rings may be seen in glories than in coronas because the decrease in luminance away from the backward direction is not as steep as that away from the forward direction. To see a glory from an airplane, look for colored rings around its shadow cast on clouds below. This shadow is not an essential part of the glory, it merely directs you to the antisolar point.

Like the rainbow, the glory may be looked upon as a singularity in the differential scattering cross section Eq. (35). Equation (36) gives one set of conditions for a singularity; the second set is

$$\sin\theta = 0, \quad b(\theta) \neq 0. \qquad (38)$$

That is, the differential scattering cross section is infinite for nonzero impact parameters (corresponding to incident rays that do not intersect the center of the sphere) that give forward (0°) or backward (180°) scattering. The forward direction is excluded because this is the direction of intense scattering accounted for by the Fraunhofer theory.

For one internal reflection, Eq. (38) leads to the condition

$$\sin\Theta_i = \frac{n}{2}\sqrt{4 - n^2}, \qquad (39)$$

which is satisfied only for refractive indices between 1.414 and 2, the lower refractive index corresponding to a grazing-incidence ray. The refractive index of water lies outside this range. Although a condition similar to Eq. (39) is satisfied for rays undergoing four or more internal reflections, insufficient energy is associated with such rays. Thus, it seems that we have reached an impasse: the theoretical condition for a glory cannot be met by water droplets. Not so, says van de Hulst [18] in a seminal paper. He argues that 1.414 is close enough to 1.33 given that geometrical optics is, after all, an approximation. Cloud droplets are large compared with the wavelength, but not so large that geometrical optics is an infallible guide to their optical behavior. Support for the van de Hulstian interpretation of glories was provided by Bryant and Cox [19], who showed that the dominant contribution to the glory is from the last terms in the exact series for scattering by a sphere. Each successive term in this series is associated with ever larger impact parameters. Thus, the terms that give the glory are indeed those corresponding to grazing rays. Further unraveling of the glory and vindication of van de Hulst's conjectures about the glory were provided by Nussenzveig [20].

It sometimes is asserted that geometrical optics is incapable of treating the glory. Yet the same can be said for the rainbow. Geometrical optics explains rainbows only in the sense that it predicts singularities for scattering in certain directions (rainbow angles). But it can predict only the angles of intense scattering, not the amount. Indeed, the error is infinite. Geometrical optics also predicts a singularity in the backward direction. Again, this simple theory is powerless to predict more. Results from geometrical optics for both rainbows and glories are not the end but rather the beginning, an invitation to take a closer look with more powerful magnifying glasses.

8 Scattering by Single Ice Crystals

Scattering by spherical water drops in the atmosphere gives rise to three distinct displays in the sky: coronas, rainbows, and glories. Ice particles (crystals) also can inhabit the atmosphere, and they introduce two new variables in addition to size: shape and orientation, the second a consequence of the first. Given this increase in the number of degrees of freedom, it is hardly cause for wonder that ice crystals are the source of a greater variety of displays than are water drops. As with rainbows, the gross features of ice-crystal phenomena can be described simply with geometrical optics, various phenomena arising from the various fates of rays incident on crystals. Colorless displays (e.g., sun pillars) are generally associated with reflected rays, colored displays (e.g., sun dogs and halos) with refracted rays. Because of the wealth of ice-crystal displays, it is not possible to treat all of them here, but one example should point the way toward understanding many of them.

8.1 Sun Dogs and Halos

Because of the hexagonal crystalline structure of ice it can form as hexagonal plates in the atmosphere. The stable position of a plate falling in air is with the normal to its face more or less vertical, which is easy to demonstrate with an ordinary business card. When the card is dropped with its edge facing downward (the supposedly aerodynamic position that many people instinctively choose), the card somersaults in a helter-skelter path to the ground. But when the card is dropped with its face parallel to the ground, it rocks back and forth gently in descent.

A hexagonal ice plate falling through air and illuminated by a low sun is like a 60° prism illuminated normally to its sides (Fig. 17). Because there is no mechanism for orienting a plate within the horizontal plane, all plate orientations in this plane are equally probable. Stated another way, all angles of incidence for a fixed plate are equally probable. Yet all scattering angles (deviation angles) of rays refracted into and out of the plate are not equally probable.

Figure 18 shows the range of scattering angles corresponding to a range of rays incident on a 60° ice prism that is part of a hexagonal plate. For angles of incidence less than about 13°, the transmitted ray is totally internally reflected in the prism. For angles of incidence greater than about 70°, the transmittance plunges. Thus, the only rays of consequence are those incident between about 13° and 70°.

All scattering angles are not equally probable. The (uniform) probability distribution $p(\theta_i)$ of incidence angles θ_i is related to the probability distribution

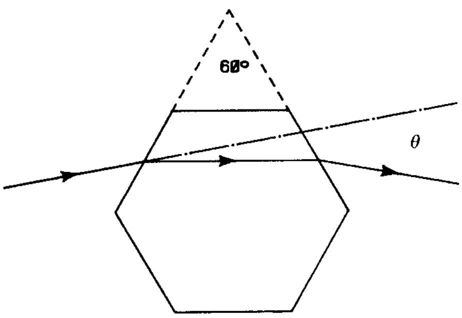

Fig. 17 Scattering by a hexagonal ice plate illuminated by light parallel to its basal plane. The particular scattering angle θ shown is an angle of minimum deviation. The scattered light is that associated with two refractions by the plate

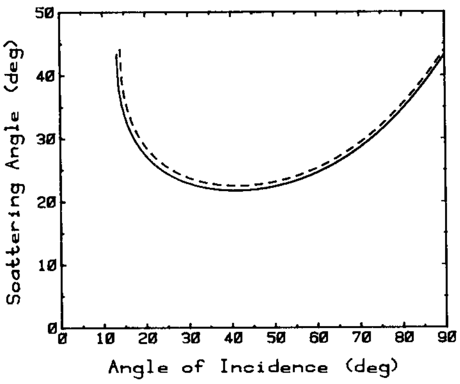

Fig. 18 Scattering by a hexagonal ice plate (see Fig. 17) in various orientations (angles of incidence). The solid curve is for red light, the dashed for blue light

$P(\theta)$ of scattering angles θ by

$$P(\theta) = \frac{p(\theta_i)}{d\theta / d\theta_i}. \tag{40}$$

At the incidence angle for which $d\theta / d\theta_i = 0$, $P(\theta)$ is infinite and scattered rays are intensely concentrated near the corresponding angle of minimum deviation.

The physical manifestation of this singularity (or caustic) at the angle of minimum deviation for a 60° hexagonal ice plate is a bright spot about 22° from either or both sides of a sun low in the sky. These bright spots are called *sun dogs* (because they accompany the sun) or *parhelia* or *mock suns*.

The angle of minimum deviation θ_m, hence the angular position of sun dogs, depends on the prism angle Δ (60° for the plates considered) and refractive index:

$$\theta_m = 2\sin^{-1}\left(n \sin\frac{\Delta}{2}\right) - \Delta. \tag{41}$$

Because ice is dispersive, the separation between the angles of minimum deviation for red and blue light is about 0.7° (Fig. 18), somewhat greater than the angular width of the sun. As a consequence, sun dogs may be tinged with color, most noticeably toward the sun. Because the refractive index of ice is least at the red end of the spectrum, the red component of a sun dog is closest to the sun. Moreover, light of any two wavelengths has the same scattering angle for different angles of incidence if one of the wavelengths does not correspond to red. Thus, red is the purest color seen in a sun dog. Away from its red inner edge a sun dog fades into whiteness.

With increasing solar elevation, sun dogs move away from the sun. A falling ice plate is roughly equivalent to a prism, the prism angle of which increases with solar elevation. From Eq. (41) it follows that the angle of minimum deviation, hence the sun dog position, also increases.

At this point you may be wondering why only the 60° prism portion of a hexagonal plate was singled out for attention. As evident from Fig. 17, a hexagonal plate could be considered to be made up of 120° prisms. For a ray to be refracted twice, its angle of incidence at the second interface must be less than the critical angle. This imposes limitations on the prism angle. For a refractive index 1.31, all incident rays are totally internally reflected by prisms with angles greater than about 99.5°.

A close relative of the sun dog is the 22° halo, a ring of light approximately 22° from the sun (Fig. 19). Lunar halos are also possible and are observed frequently (although less frequently than solar halos); even moon dogs are possible. Until Fraser [21] analyzed halos in detail, the conventional wisdom had been that they obviously were the result of randomly oriented crystals, yet another example of jumping to conclusions. By combining optics and aerodynamics, Fraser showed that if ice crystals are small enough to be randomly oriented by Brownian motion, they are too small to yield sharp scattering patterns.

But completely randomly oriented plates are not necessary to give halos, especially ones of nonuniform brightness. Each part of a halo is contributed to by plates with a different tip angle (angle between the normal to the plate and the vertical). The transition from oriented plates (zero tip angle) to randomly oriented plates occurs over a narrow range of sizes. In the transition region, plates can be small enough to be partially oriented yet large enough to give a distinct contribution to the halo. Moreover, the mapping between tip angles and azimuthal angles on the halo depends on solar elevation. When the sun is near the horizon, plates can give a distinct halo over much of its azimuth.

Fig. 19 A 22° solar halo. The hand is not for artistic effect but rather to occlude the bright sun

When the sun is high in the sky, hexagonal plates cannot give a sharp halo but hexagonal columns – another possible form of atmospheric ice particles – can. The stable position of a falling column is with its long axis horizontal. When the sun is directly overhead, such columns can give a uniform halo even if they all lie in the horizontal plane. When the sun is not overhead but well above the horizon, columns also can give halos.

A corollary of Fraser's analysis is that halos are caused by crystals with a range of sizes between about 12 and 40 μm. Larger crystals are oriented; smaller particles are too small to yield distinct scattering patterns.

More or less uniformly bright halos with the sun neither high nor low in the sky could be caused by mixtures of hexagonal plates and columns or by clusters of bullets (rosettes). Fraser opines that the latter is more likely.

One of the by-products of his analysis is an understanding of the relative rarity of the 46° halo. As we have seen, the angle of minimum deviation depends on the prism angle. Light can be incident on a hexagonal column such that the prism angle is 60° for rays incident on its side or 90° for rays incident on its end. For $n = 1.31$, Eq. (41) yields a minimum deviation angle of about 46° for $\Delta = 90°$. Yet, although 46° halos are possible, they are seen much less frequently than 22° halos. Plates cannot give distinct 46° halos although columns can. Yet they must be solid and most columns have hollow ends. Moreover, the range of sun elevations is restricted.

Like the green flash, ice-crystal phenomena are not intrinsically rare. Halos and sun dogs can be seen frequently – once you know what to look for. Neuberger [22] reports that halos were observed in State College, Pennsylvania, an average of 74 days a year over a 16-year period, with extremes of 29 and 152 halos a year. Although the 22° halo was by far the most frequently seen display, ice-crystal displays of all kinds were seen, on average, more often than once every four days at a location not especially blessed with clear skies. Although thin clouds are necessary for ice-crystal displays, clouds thick enough to obscure the sun are their bane.

9
Clouds

Although scattering by isolated particles can be studied in the laboratory, particles in the atmosphere occur in crowds (sometimes called *clouds*). Implicit in the previous two sections is the assumption that each particle is illuminated solely by incident sunlight; the particles do not illuminate each other to an appreciable degree. That is, clouds of water droplets or ice grains were assumed to be optically thin, and hence multiple scattering was negligible. Yet the term cloud evokes fluffy white objects in the sky, or perhaps an overcast sky on a gloomy day. For such clouds, multiple scattering is not negligible, it is the major determinant of their appearance. And the quantity that determines the degree of multiple scattering is optical thickness (see Sec. 2.4).

9.1
Cloud Optical Thickness

Despite their sometimes solid appearance, clouds are so flimsy as to be almost nonexistent – except optically. The fraction of the total cloud volume occupied by water substance (liquid or solid) is about 10^{-6} or less. Yet although the mass density of clouds is that of air to within a small fraction of a percent, their optical thickness (per unit physical thickness) is much greater. The number density of air molecules is vastly greater than that of water droplets in clouds, but scattering per molecule of a cloud droplet is also much greater than scattering per air molecule (see Fig. 7).

Because a typical cloud droplet is much larger than the wavelengths of visible light, its scattering cross section is to good approximation proportional to the square of its diameter. As a consequence, the scattering coefficient [see Eq. (2)] of a cloud having a volume fraction f of droplets is approximately

$$\beta = 3f\frac{\langle d^2 \rangle}{\langle d^3 \rangle}, \tag{42}$$

where the brackets indicate an average over the distribution of droplet diameters d. Unlike molecules, cloud droplets are distributed in size. Although cloud particles can be ice particles as well as water droplets, none of the results in this and the following section hinge on the assumption of spherical particles.

The optical thickness along a cloud path of physical thickness h is βh for a cloud with uniform properties. The ratio $\langle d^3 \rangle / \langle d^2 \rangle$ defines a mean droplet diameter, a typical value for which is 10 μm. For this diameter and $f = 10^{-6}$, the optical thickness per unit meter of physical thickness is about the same as the normal optical thickness of the atmosphere in the middle of the visible spectrum (see Fig. 3). Thus, a cloud only 1 m thick is equivalent optically to the entire gaseous atmosphere.

A cloud with (normal) optical thickness about 10 (i.e., a physical thickness of about 100 m) is sufficient to obscure the disk of the sun. But even the thickest cloud does not transform day into night. Clouds are usually translucent, not transparent, yet not completely opaque.

The scattering coefficient of cloud droplets, in contrast with that of air molecules, is more or less independent of wavelength. This is often invoked as the cause of the colorlessness of clouds. Yet wavelength independence of scattering by a single particle is only sufficient, not necessary, for wavelength independence of scattering by a cloud of particles (see Sec. 2.4). Any cloud that is optically thick and composed of particles for which absorption is negligible is white upon illumination by white light. Although absorption by water (liquid and solid) is not identically zero at visible wavelengths, and selective absorption by water can lead to observable consequences (e.g., colors of the sea and glaciers), the appearance of all but the thickest clouds is not determined by this selective absorption.

Equation (42) is the key to the vastly different optical characteristics of clouds and of the rain for which they are the progenitors. For a fixed amount of water (as specified by the quantity fh), optical thickness is inversely proportional to mean diameter. Rain drops are about 100 times larger on average than cloud droplets, and hence optical thicknesses of rain shafts are correspondingly smaller. We often can see through many kilometers of intense rain whereas a small patch of fog on a well-traveled highway can result in carnage.

9.2
Givers and Takers of Light

Scattering of visible light by a single water droplet is vastly greater in the forward ($\theta < 90°$) hemisphere than in the backward ($\theta > 90°$) hemisphere (Fig. 9). But water droplets in a thick cloud illuminated by sunlight collectively scatter much more in the backward hemisphere (reflected light) than in the forward hemisphere (transmitted light). In each scattering event, incident photons are deviated, on average, only slightly, but in many scattering events most photons are deviated enough to escape from the upper boundary of the cloud. Here is an example in which the properties of an ensemble are different from those of its individual members.

Clouds seen by passengers in an airplane can be dazzling, but if the airplane were to descend through the cloud these same passengers might describe the cloudy sky overhead as gloomy. Clouds are both givers and takers of light. This dual role is exemplified in Fig. 20, which shows the calculated diffuse downward irradiance below clouds of varying optical thickness. On an airless planet the sky would be black in all directions (except directly toward the sun). But if the sky were to be filled from horizon to horizon with a thin cloud, the brightness overhead would markedly increase. This can be observed in a partly overcast sky, where gaps between clouds (blue sky) often are noticeably darker than

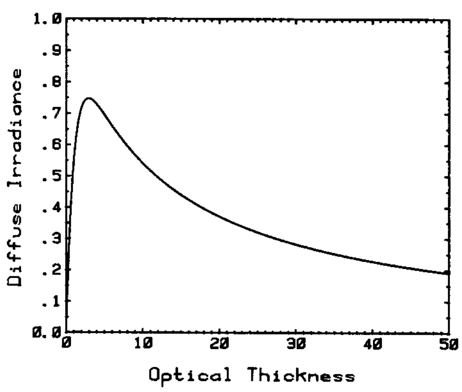

Fig. 20 Computed diffuse downward irradiance below a cloud relative to the incident solar irradiance as a function of cloud optical thickness

their surroundings. As so often happens, more is not always better. Beyond a certain cloud optical thickness, the diffuse irradiance decreases. For a sufficiently thick cloud, the sky overhead can be darker than the clear sky.

Why are clouds bright? Why are they dark? No inclusive one-line answers can be given to these questions. Better to ask, Why is that particular cloud bright? Why is that particular cloud dark? Each observation must be treated individually; generalizations are risky. Moreover, we must keep in mind the difference between brightness and radiance when addressing the queries of human observers. Brightness is a sensation that is a property not only of the object observed but of its surroundings as well. If the luminance of an object is appreciably greater than that of its surroundings, we call the object bright. If the luminance is appreciably less, we call the object dark. But these are relative rather than absolute terms.

Two clouds, identical in all respects, including illumination, may still appear different because they are seen against different backgrounds, a cloud against the horizon sky appearing darker than when seen against the zenith sky.

Of two clouds under identical illumination, the smaller (optically) will be less bright. If an even larger cloud were to appear, the cloud that formerly had been described as white might be demoted to gray.

With the sun below the horizon, two identical clouds at markedly different elevations might appear quite different in brightness, the lower cloud being shadowed from direct illumination by sunlight.

A striking example of dark clouds can sometimes be seen well after the sun has set. Low-lying clouds that are not illuminated by direct sunlight but are seen against the faint twilight sky may be relatively so dark as to seem like ink blotches.

Because dark objects of our everyday lives usually owe their darkness to absorption, nonsense about dark clouds is rife: they are caused by pollution or soot. Yet of all the reasons that clouds are sometimes seen to be dark or even black, absorption is not among them.

Glossary

Airlight: Light resulting from scattering by all atmospheric molecules and particles along a line of sight.

Antisolar Point: Direction opposite the sun.

Astronomical Horizon: Horizontal direction determined by a bubble level.

Brightness: The attribute of sensation by which an observer is aware of differences of luminance (definition recommended by the 1922 Optical Society of America Committee on Colorimetry).

Contrast Threshold: The minimum relative luminance difference that can be perceived by the human observer.

Inferior Mirage: A mirage in which images are displaced downward.

Irradiance: Radiant power crossing unit area in a hemisphere of directions.

Lapse Rate: The rate at which a physical property of the atmosphere (usually temperature) decreases with height.

Luminance: Radiance integrated over the visible spectrum and weighted by the spectral response of the human observer. Also sometimes called *photometric brightness*.
.5.5

Mirage: An image appreciably different from what it would be in the absence of atmospheric refraction.

Neutral Point: A direction in the sky for which the light is unpolarized.

Normal Optical Thickness: Optical thickness along a radial path from the surface of the earth to infinity.

Optical Thickness: The thickness of a scattering medium measured in units of photon mean free paths. Optical thicknesses are dimensionless.

Radiance: Radiant power crossing a unit area and confined to a unit solid angle about a particular direction.

Scale Height: The vertical distance over which a physical property of the atmosphere is reduced to $1/e$ of its value.

Scattering Angle: Angle between incident and scattered waves.

Scattering Coefficient: The product of scattering cross section and number density of scatterers.

Scattering Cross Section: Effective area of a scatterer for removal of light from a beam by scattering.

Scattering Plane: Plane determined by incident and scattered waves.

Solar Point: The direction toward the sun.

Superior Mirage: A mirage in which images are displaced upward.

Tangential Optical Thickness: Optical thickness through the atmosphere along a horizon path.

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Further Reading

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