ME 457    Experimental Solid Mechanics (Lab)

Torsion Test : Solid and Hollow Shafts

Introduction
The purpose of torsion testing usually parallels that of uniaxial tension tests. From the experiment, the shear elastic modulus (G), shear proportional stress (τ_p), shear yield stress (τ_y), and the stress-strain behavior in general, can be obtained. However, in contrast to uniaxial tension tests, the stresses are not distributed uniformly over the cross section.

Each test will be conducted until failure i.e., it will end in the buckling of the hollow specimen or fracture for the solid specimen. Thus, this experiment will also allow observing the different modes of failure for solid and hollow circular shafts made of ductile or brittle materials.

Torsion loading results in twisting of one section of a body with respect to a contiguous section. During the test the angle of twist (ϕ) and the applied torque (T) are measured as the test proceeds. For a circular cross-section, in the absence of the other loads, pure shear stress state exists at each point. Torsional elastic shear stresses vary linearly from zero at the axis of twist to a maximum at the extreme fibers. Thus, in a solid circular bar, when the surface fibers reach the yield shear stress, they are, in a sense, supported by elastic interior fibers. Consequently, the elastic resistance of the remainder of the section masks the effect of yielding of the surface fibers during their early stage of yielding. Usually, it is not until considerable yielding has taken place that any noticeable effect of nonlinearity is apparent using a simple mechanical troptometer to measure the angle of twist (ϕ) (calibrated in increments of 0.2 degrees). Therefore, it is practically impossible to determine when the extreme fibers of the solid specimen in torsion start to yield. This difficulty is overcome by the use of hollow (thin-walled) specimens, which give more sensitive measures of the elastic-plastic transition since all the fibers are at about the same stress. However, for thin-walled tubes with large ratios of diameter vs. thickness (D/t > 10) there is a tendency for premature local buckling failure to occur. Therefore the actual dimensions of the specimen used must be carefully chosen.

To test the material in torsion the proper test procedure is needed. It involves mounting a shaft into the testing machine, applying torque incrementally and measuring both the applied torque and the corresponding angle of twist. Using the appropriate formulae, relationships and the measured dimensions, we can determine the shear stress and shear strain on the shaft. Then, one can plot the torque vs. angle of twist, and shear stress vs. shear strain from which one can find the material properties previously mentioned.

The assumptions made in this experiment include but are not limited to the following:
1. The torque is applied along the center of axis of the shaft.
2. The material is tested at steady state (absence of strain rate effects).
3. Plane sections remain plane after twisting (the circular section conforms to this condition).
Apparatuses:
1. Torsion testing specimens:
   - Solid (ductile) aluminum shaft,
   - Hollow (ductile) aluminum shaft *(both ductile specimens are made of the same material)*,
   - Solid (brittle) aluminum shaft.
2. Caliper
3. Troptometer
4. Torsion testing machine
5. Measuring tape
6. Safety glasses

Test Procedure *(for solid and hollow ductile shafts):*
1. Measure the diameter of the test specimen using the caliper (take an average of 5 measurements).
2. Slide the shaft into the troptometer and tighten down the two fastening screws to the shaft.
3. If the hollow shaft of aluminum is used, slide two steel inserts into the ends where the sample will be clamped down.
4. Clean the clamps used to hold the shaft in place.
5. Insert the shaft into the right clamps’ chucks in the testing machine. Tighten the clamp ensuring that the grip is very tight.
6. Slide the other clamp over the other end of the shaft, and tighten down tightly as well.
7. Adjust the troptometer to the correct position from both ends *(i.e. do not let it contact with any part of the testing machine.)*.
8. Measure the length between the troptometer clamps $L_t$ using the tape measure. This length will be used in calculations later.
9. Next, measure the length between the torsion machine's clamps, $L_c$.

*Note: While inserting the specimen into the clamps, move the weighing house to the right until close to or stopped by the shaft end bearing against the inside of the chuck.*

10. Make zero the torque indicator by aligning the red needle with the black one on the torque indicator. Also, use the zeroing lever to make sure that the needles point to zero.
11. Make zero the troptometer indicator by hand.
12. Turn the crank manually forwards (counter-clockwise) and observe the troptometer indicator. You want to increase the angle of twist by $1^\circ$ increments up to $12^\circ$.
13. At each interval as describe in the above step, read the measurement (using the red needle) displayed by the torque gage. **Note:** The torque gage has to be read from 0 to 10,000 in-lb. Hence, every reading that you get on the regular scale must be multiplied by 10 for the correct results.
14. Also take the reading off the scale on the right (rotating) clamp housing. This should be done at every even degree that is reached by the troptometer *(i.e. at 2°, 4°, 6° etc.)*
15. Construct the following table and tabulate the torque $T$, and the corresponding angle of twist of the troptometer $\phi_t$, and clamp $\phi_c$: 
16. Once 12° is reached, unload the specimen by turning the crank backwards (clockwise) and take the torque gage reading and the angle of twist of the clamp at 1° intervals (read off the black needle this time). All the while, watch the torque gage making sure that it has not reached zero yet. Tabulate this data as a continuation on the same table used in step 15.

17. Watch for the torque gage to read zero. At this point, take down the troptometer reading and the angle of twist of the clamp.

18. Next reload the specimen by turning the crank manually forwards (counterclockwise) and observe the troptometer indicator. You want to increase the angle of twist by 1° increments up to the final angle of 20°.

19. At each interval as describe in the above step, read the measurement (using the red needle) displayed by the torque gage. **NOTE: The torque gage has to be read from 0 to 10,000 in-lb. Hence, every reading that you get on the regular scale must be multiplied by 10 for the correct results**

20. When \( \phi_t = 20^\circ \) totally loosen the troptometer and let it hang loosely and twist the specimen to failure. The hollow aluminum shaft is considered to have failed when it buckles.

Note the angle of twist that was needed to fracture or buckle the specimen from the angle of twist indicator located on the right rotating arm of the torsion machine, \( \phi_c \).

It is best to count the number of rotations and then later convert it to degrees when writing the report. Also, record the final torque reading of the specimen at failure, \( T_f \).

**Test Procedure (for a brittle shaft):**

1. Insert the brittle (cast aluminum) shaft into the machine's clamps and tighten them. Twist the specimen until failure and observe the failure mode.

**Analysis:**

1. For two ductile shafts (hollow and solid) complete the above table by using the following equations. Use the torque and the troptometer angles of twist \( \phi_t \) \textit{(not the angle of twist of the clamp \( \phi_c \), it will be used later to calculate the fracture strain \( \gamma_f = (\phi_c)/(r/L_c) \)):}

   (a) \( \tau = \frac{T}{r/l_p} \) - valid only in the elastic range, for both solid and hollow shafts.

   (b) \( \tau_{avg} = \frac{T}{2\pi(r_{avg})^2} \) - valid only for the hollow shaft in the elastic and inelastic ranges.

   (c) \( \gamma = \frac{\phi_t}{r/L_t} \) - valid in the elastic and inelastic ranges for both shafts.
In order to find the actual shear stress for the inelastic loading portion of the solid shaft (since there are no equations to calculate them), continue on with step 2 and plot, first the shear stress vs. shear strain diagram for the hollow shaft. Using this plot, cross-referenced the shear stress from the calculated shear strains attained from the solid shaft data (see hint on page 5). We are able to do this because the stresses are nearly uniform throughout the wall thickness of the hollow shaft as compared to the solid one (both shafts are made of the same material). The cross-referenced shear stress can then be tabulated in your table as the “actual” shear stresses at the surface of the solid shaft.

2. Plot the graphs:
   • Torque (ordinate) vs. angle of twist (abscissa)
   • Shear stress (ordinate) vs. shear strain (abscissa)
   There should be 4 graphs in all, two for each specimen.

3. From the graphs determine:
   • Proportional shear stress ($\tau_p$) for the hollow shaft.
   • Yield shear stress based on a 0.2% offset ($\tau_y$) for the hollow shaft.
   • Shear modulus of elasticity ($G$) during loading and unloading for both shafts.
   • Residual shear strain ($\gamma_r$) for both shafts
   • Next, starting at the point of residual strain $\gamma_r$, calculate the new proportional limit ($\tau_p$), yield point ($\tau_y$) based on a 0.2% offset, shear modulus of elasticity ($G$). In addition, calculate the fracture strain ($\gamma_f$) and corresponding fracture strength ($\tau_f$) in torsion. Compare and discuss in detail the differences between these new: $\tau_p$, $\tau_y$, and $G$ values and those obtained in the beginning of the torsion test experiment. Explain any observed differences concisely.

Note:
• The point of inflection at the end of the elastic portion of the loading curve is where $\tau_p$ is located.
• Where the loading curve meets the 0.2% offset line is where you will find $\tau_y$.
• The shear elastic modulus, $G$, usually is not obtained as the shear stress, $\tau$, divided by the corresponding shear strain, $\gamma$, but rather the slope of the linear portion of the stress-strain ($\tau$-$\gamma$) curve. This slope is calculated by dividing the change of shear stress, $\Delta \tau$, by the corresponding change of shear strain, $\Delta \gamma$, in each linear range of loading and unloading case from the stress-strain curves, i.e. $G = \Delta \tau / \Delta \gamma$. The shear stress vs. shear strain graph should have two curves: one corresponding to the initial loading up to 12° with subsequent unloading to zero torque and the other reloading curve which starts from the residual shear strain ($\gamma_r$) up to 20° and then to failure, both drawn on the same plot (vary them by color or some other means).
• The residual shear strain, $\gamma_r$, is determined by measuring the strain between the zero shear strain and the value after unloading to zero torque. (For optional extra credit: the residual shear stress in the outside fibers of the solid shaft, $\tau_r$, can be estimated as: $\tau_r = \tau_{max} - \Delta \tau$, where due to elastic unloading $\Delta \tau = \Delta T r/I_p = T_{max} r/I_p$.)
4. Discuss the failure modes for all three specimens based on the relation between pure shear and normal stresses (see the helpful figure below).
   a) Explain why, in torsion test, the ductile solid shaft sheared on a right section whereas the brittle (cast aluminum) solid shaft fractured on a helicoidal path.
   b) Explain why the brittle cast aluminum specimen fractured at a “negative-slope” 45° angle while the ductile hollow aluminum specimen buckled at a “positive-slope” 45° angle. (The figure below should be a good hint.)
   c) Why did the hollow specimen experience local-buckling (the twisting shape at failure) under torsion? What would happen to the twisting angles of local-buckling if the torsion was done in the opposite direction?

![Diagram showing pure shear and normal stresses](image)

5. It was observed that we had to use cross-referenced shear stresses from the hollow shaft \( \tau vs. \gamma \) curve to predict the “actual” stresses on the surface of solid shaft during inelastic loading. We want to investigate the difference between this “actual” shear stress and the one we would get from equation (a) in step 1 above. Plot the shear stress vs. shear strain graphs for the ductile solid shaft again but this time draw two curves: one based on the cross-referenced shear stresses and the other based on the shear stresses obtained from equation (a) of step 1 (see hint below). Compare the two and give a thorough analysis on why there is a difference between them.

**Hint:** How the cross-referenced shear stresses can be determined.

![Diagram showing cross-referencing](image)