SUBSPACE-BASED CHANNEL ESTIMATION FOR WIDEBAND CDMA COMMUNICATION SYSTEMS

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ABSTRACT

Subspace-based channel estimation for CDMA systems exploits the eigenstructure of the received data matrix, allowing reconstruction of signature waveforms without the knowledge of input signals. In this context, we consider the estimation of multiuser channels for wideband CDMA systems. By relying on knowledge of the structure of the propagation channel, the proposed channel identification is computationally more efficient. In particular, we reduce the number of unknown parameters in the new channel model by forming a set of spatial parameters and a set of temporal parameters. Simulations are performed to evaluate the algorithm.

I. Introduction

In wideband CDMA (W-CDMA) systems, the intersymbol interference (ISI) introduced by multipath propagation can be significant due to the large multipath delay spread. ISI, together with the user asynchronism in the reverse link, distorts the code orthogonality. Therefore, explicit knowledge of all user channels is needed for equalization and multiuser detection. One of the key features of W-CDMA systems is the pilot-assisted channel estimation, in which a large portion of the bandwidth is reserved for the training sequence overhead. If the pilots are inserted sparsely in the transmitted data stream, the performance of channel estimation is limited in a fast-fading environment.

Alternatively, blind channel estimation offers better spectrum efficiency, and has received increasing attention [1], [2], [3], [4], [5]. By assuming knowledge of the user codes but no training sequences, these subspace-based blind techniques model each of the user channels as an FIR filter and identify it by exploiting the second-order statistics of the channel output. In W-CDMA, the order of an FIR filter identifying the entire user channel can be quite large.

Improved performance in statistical estimation can be expected if one can put enough constraint on the model structure. Modifications have been made in [6] to reduce the number of unknown parameters, where the knowledge of the structure of the propagation channel is incorporated in the subspace methods. However, an extensive multidimensional optimization search is required in this parametric subspace approach.

In this paper, we propose a deterministic approach to estimate the user channels in W-CDMA systems. Taking into account the *a priori* information of the structure of the propagation channel, we describe the channel by a reduced number of model parameters, i.e. two separable sets of spatial and temporal parameters. Our approach is computationally more efficient than the traditional subspace method on the unstructured channel. The nominal path arrival angles and delays, which describe the general propagation structure, vary slowly compared to the multipath fading rate. They can be estimated and tracked using a small number of pilot symbols over a long period of time. In the following sections, the reverse link of the W-CDMA communication systems is considered.

II. Analytical Model of W-CDMA Systems

A. Structured Channel Model

Consider the case of a mobile user communicating with a base station (BS) with an array of M antenna elements. The baseband multipath fading vector channel response can be described as [7]

$$\mathbf{h}(t) = \sum_{i=1}^{P} \mathbf{a}(\theta_i)\beta_i p(t-\tau_i) \qquad (1)$$

where P is the total number of paths. $p(t)$ is the chip pulse shaping filter. $\mathbf{a}(\theta_i)$, β_i and τ_i are the array response, the complex amplitude and the delay associated with the i^{th} path, respectively. In W-CDMA systems, the multipath delay spread may span several symbol periods.

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In the reverse link, the signals transmitted by the mobile users are first scattered by objects local to the mobile, and then the scattering ray bundles are reflected or diffracted by objects remote to the mobile. Within each bundle of rays from a remote reflector, the scattering rays come with arrival angles close to the nominal direction-of-arrival (DOA) and arrive at the BS with a small delay spread. The delay difference between ray bundles are relatively large. Assume that the nominal delays of ray bundles are separable by the W-CDMA receiver. This propagation structure causes BS path arrivals to cluster in a 2-D direction-of-arrival (DOA) vs. time-of-arrival (TOA) plot.

Suppose there are Q DOA-TOA clusters viewed by the BS. The channel response corresponding to the q^{th} cluster can be described as

$$\mathbf{h}_q(t) = \sum_{k=1}^{P_q} \mathbf{a}(\theta_q + \tilde{\theta}_{qk})\beta_{qk} p(t - \tau_{qk}) \quad (2)$$

where P_q is the number of scattering paths within. $\theta_g + \tilde{\theta}_{qk}$ is the arrival angel with nominal DOA θ_q. With $|\tilde{\theta}_{qk}|$ being small, a first-order Taylor series expansion of (2) yields (we drop the subscript q on the right side)

$$\begin{aligned} \mathbf{h}_q(t) &\simeq \sum_k (\mathbf{a}(\theta) + \tilde{\theta}_k \mathbf{d}(\theta))\beta_k p(t - \tau_k) \\ &= \mathbf{a}(\theta) \sum_k \beta_k p(t - \tau_k) + \mathbf{d}(\theta) \sum_k \tilde{\theta}_k \beta_k p(t - \tau_k) \\ &= \mathbf{a}(\theta) g_1(t - \tau) + \mathbf{d}(\theta) g_2(t - \tau) \quad (3) \end{aligned}$$

where $\mathbf{d}(\theta) = \partial \mathbf{a}(\theta)/\partial\theta$ is the gradient. Therefore the entire vector channel response follows as

$$\mathbf{h}(t) = \sum_{q=1}^{2Q} \mathbf{a}_q \; g_q(t - \tau_q) \quad (4)$$

where, each path cluster as a dispersive channel is modeled as two FIR filters $g_{2i-1}(t)$ and $g_{2i}(t)$ $(i = 1, \ldots, Q)$, with short supports compared to the symbol period. $\{\tau_q\}$ are the nominal delays and are assumed to be pre-estimated by the BS. Note that because $\{g_q(t)\}$ are somewhat arbitrary, the nominal delays do not need to be estimated precisely, and they can be easily tracked when channel varies. We process $\{\mathbf{a}_q\}$ as arbitrary vectors.

B. Data Model

In CDMA systems, the transmitted symbols are spread to chip-rate data by the user-specified spreading code of length L_c. The signal received at BS antenna array is $\mathbf{x}(t) = \mathbf{w}(t) * s(t)$, where $s(t)$ is the transmitted signal. Vector $\mathbf{w}(t)$ is the user signature waveform, which is the convolution of the spreading code and the channel response $\mathbf{w}(t) = c(t) * \mathbf{h}(t)$. We introduce the notation for the discrete-time data sampled at the chip rate

$$\begin{aligned} \bar{\mathbf{x}}(k) &= [\mathbf{x}^T(kL_c - L_c + 1) \; \cdots \; \mathbf{x}^T(kL_c)]^T \quad (5) \\ \bar{\mathbf{h}} &= [\mathbf{h}_1^T \; \mathbf{h}_2^T \; \cdots \; \mathbf{h}_{L_h}^T]^T \quad (6) \\ \bar{\mathbf{w}} &= [\mathbf{w}_1^T \; \mathbf{w}_2^T \; \cdots \; \mathbf{w}_{L_c+L_h-1}^T]^T \quad (7) \end{aligned}$$

where L_h is the channel length of $\mathbf{h}(t)$. The signature waveform $\bar{\mathbf{w}}$ can be expressed as

$$\bar{\mathbf{w}} = \underbrace{\begin{bmatrix} c_1 & & \\ \vdots & \ddots & \\ c_{L_c} & & c_1 \\ & \ddots & \vdots \\ & & c_{L_c} \end{bmatrix} \otimes \mathbf{I}_M}_{\mathbf{C}} \; \bar{\mathbf{h}} \quad (8)$$

where $\{c_k, \; k = 1, 2, \ldots, L_c\}$ is the spreading code, and $\otimes$ denotes the Kronecker product. From (4), the discrete vector channel can be expressed as

$$\bar{\mathbf{h}} = \mathbf{G} \; \bar{\mathbf{a}} \quad (9)$$

where $\bar{\mathbf{a}} = [\mathbf{a}_1^T \cdots \mathbf{a}_{2Q}^T]^T$ and $\mathbf{G} = \mathcal{G} \otimes \mathbf{I}_M$. $\mathcal{G}$ is a $L_h \times 2Q$ matrix whose columns are defined as

$$\mathrm{Col}^{(q)}\{\mathcal{G}\} = \underbrace{\begin{bmatrix} \mathbf{0}_{\mathrm{I}} \\ --- \\ \mathbf{I}_{L_g} \\ --- \\ \mathbf{0}_{\mathrm{II}} \end{bmatrix}}_{\mathcal{F}_q} \underbrace{\begin{bmatrix} g_q(1) \\ g_q(2) \\ \vdots \\ g_q(L_g) \end{bmatrix}}_{\mathbf{g}_q} \quad (10)$$

where $\mathbf{0}_{\mathrm{I}}$ and $\mathbf{0}_{\mathrm{II}}$ are zero matrices with dimensions $l_q \times L_g$ and $(L_h - L_g - l_q) \times L_g$, respectively. l_q is the known nominal delay index. L_g is the maximum order of FIR filters $\{\mathbf{g}_d, \; d = 1, \ldots, 2Q\}$.

The vector channel from (4) can also be expressed as

$$\bar{\mathbf{h}} = \mathbf{A} \; \bar{\mathbf{g}} \quad (11)$$

where $\bar{\mathbf{g}} = [\mathbf{g}_1^T \cdots \mathbf{g}_{2Q}^T]^T$ and $\mathbf{A}$ is a $ML_h \times 2QL_g$ matrix defined as

$$\mathbf{A} = [\mathcal{F}_1 \otimes \mathbf{a}_1, \; \mathcal{F}_2 \otimes \mathbf{a}_2, \cdots, \mathcal{F}_{2Q} \otimes \mathbf{a}_{2Q}] \quad (12)$$

Suppose signature waveform $\bar{\mathbf{w}}$ spans L symbols, i.e. $L = \lceil \frac{L_h - 1}{L_c} \rceil + 1$, we partition vector $\bar{\mathbf{w}}$ into L parts, each having length ML_c. If the last one does not have sufficient elements, it is padded with zeros. Each part is denoted as

$$\bar{\mathbf{w}}_k = [\mathbf{w}_{kL_c-Lc+1}^T \;\; \mathbf{w}_{kL_c-Lc+2}^T \;\; \cdots \;\; \mathbf{w}_{kL_c}^T]^T \tag{13}$$

Therefore the noiseless baseband signal received by the BS over a symbol period sampled at the chip rate is

$$\bar{\mathbf{x}}(k) = [\bar{\mathbf{w}}_L \; \bar{\mathbf{w}}_{L-1} \cdots \bar{\mathbf{w}}_1] \begin{bmatrix} s(k-L+1) \\ s(k-L+2) \\ \vdots \\ s(k) \end{bmatrix} \tag{14}$$

where $\{s(k)\}$ are the information bearing symbols, which belong to a finite alphabet. Collecting data $\bar{\mathbf{x}}(k)$ from N consecutive symbol periods, we form a Hankel matrix as

$$\mathbf{X} = \begin{bmatrix} \bar{\mathbf{x}}(1) & \bar{\mathbf{x}}(2) & \cdots & \bar{\mathbf{x}}(N-K+1) \\ \bar{\mathbf{x}}(2) & \cdots & \cdots & \bar{\mathbf{x}}(N-K+2) \\ \vdots & & & \vdots \\ \bar{\mathbf{x}}(K) & \cdots & \cdots & \bar{\mathbf{x}}(N) \end{bmatrix} = \mathbf{WS} \tag{15}$$

where $K(< N)$ is defined as the *smoothing factor*, and

$$\mathbf{W} = \begin{bmatrix} \bar{\mathbf{w}}_L & \cdots & \bar{\mathbf{w}}_1 & & & \\ & \bar{\mathbf{w}}_L & \cdots & \bar{\mathbf{w}}_1 & & \\ & & \ddots & & \ddots & \\ & & & \bar{\mathbf{w}}_L & \cdots & \bar{\mathbf{w}}_1 \end{bmatrix}_{ML_cK\times(K+L-1)} \tag{16}$$

$$\mathbf{S} = \begin{bmatrix} s(-L+2) & \cdots & s(N-K-L+2) \\ s(-L+3) & \cdots & s(N-K-L+3) \\ \vdots & & \vdots \\ s(K) & \cdots & s(N) \end{bmatrix} \tag{17}$$

III. Subspace-Based Channel Estimation

In the presence of additive white noise, the data matrix in (15) becomes

$$\mathbf{X} = \mathbf{WS} + \mathbf{N} \tag{18}$$

We construct the data matrix by choosing K as $\frac{L-1}{ML_c-1} < K < \frac{N-L}{2} + 1$, such that $\mathbf{W}$ is a tall matrix and $\mathbf{S}$ is a wide matrix. Due to channel independence and the randomness of symbol sequences, it is safe to assume that $\mathbf{W}$ is of full column rank and $\mathbf{S}$ is of full row rank. Perform singular value decomposition (SVD) on $\mathbf{X}$ as

$$\mathbf{X} = [\mathbf{U}_s \; \mathbf{U}_n] \begin{bmatrix} \mathbf{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^H \\ \mathbf{V}_n^H \end{bmatrix} \tag{19}$$

where $\mathbf{U}_s$ is a $ML_cK \times (K+L-1)$ matrix, and $\mathbf{U}_n$ is a $ML_cK \times (ML_cK - K - L + 1)$ matrix. The orthonormal vectors in $\mathbf{U}_s$ associated with the signal eigenvalues span the *signal subspace*, which is also the column space of $\mathbf{W}$, whereas the orthonormal vectors in $\mathbf{U}_n$ associated with the noise eigenvalues span the *noise subspace*. Due to the orthogonality between the signal subspace and the noise subspace, the columns of $\mathbf{W}$ are orthogonal to any vector in the noise subspace. Since the matrix $\mathbf{W}$ depends linearly on $\bar{\mathbf{w}}$, we have

$$\mathbf{U}_n \perp \mathbf{W} \Rightarrow \;\; \mathcal{U}_n^H \bar{\mathbf{w}} = \mathbf{0} \tag{20}$$

where $\mathcal{U}_n = [\mathcal{T}(\mathbf{u}_1) \; \cdots \; \mathcal{T}(\mathbf{u}_r)]$ with $r = ML_cK - K - L + 1$. $\mathcal{T}(\mathbf{u}_i)$ is a $ML_cL \times (K+L-1)$ block-Hankel matrix. $\mathbf{u}_i$ is the i^{th} column of $\mathbf{U}_n$ and is partitioned into K blocks with each being a vector of length ML_c. Following (8), (9) and (11), the linear equation (20) can be written as

$$\mathcal{U}_n^H \mathbf{CA}\bar{\mathbf{g}} = \mathbf{0} \tag{21}$$

$$\text{or} \qquad \mathcal{U}_n^H \mathbf{CG}\bar{\mathbf{a}} = \mathbf{0} \tag{22}$$

We choose the array response $\mathbf{a}(\theta_i)$ and its gradient $\mathbf{d}(\theta_i)$ from a point source for the nominal DOA θ_i as the initial values for $\mathbf{a}_{2i-1}$ and $\mathbf{a}_{2i}$ $(i = 1, \ldots, Q)$. Under the non-triviality constraint $||\bar{\mathbf{g}}|| = 1$, (21) can be solved for fixed $\{\mathbf{a}_q\}$ in the least-square sense, i.e. $\hat{\bar{\mathbf{g}}}$ is the normalized eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{A}^H\mathbf{C}^H\mathcal{U}_n\mathcal{U}_n^H\mathbf{CA}$. Substituting $\hat{\bar{\mathbf{g}}}$ into matrix $\mathbf{G}$, we can obtain $\{\mathbf{a}_i\}$ as the least-square solution of (22), under the constraint $||\bar{\mathbf{a}}|| = 1$. Thus we have identified $\bar{\mathbf{a}}$ and $\bar{\mathbf{g}}$, and can construct the signature waveform matrix $\mathbf{W}$ up to a phase ambiguity.

In the algorithm development, we assumed that L_g, the maximum order of FIR filters $\{g_q(t)\}$, and the channel length L_h are known *a priori*. In practice, one may select the maximum local scattering channel spread value for L_g in a particular application [7], and overestimate the channel order as $\hat{L}_h$. Therefore, the subspace spanned by $\hat{\mathbf{U}}_n$ is included in the (true) noise subspace, and the linear equation (20) is still established.

The extension of the algorithm to multi-user systems is straight forward. Suppose there are R active mobile users in the BS cell, we can rewrite (18) as

$$\mathbf{X} = \mathbf{W\Gamma S} + \mathbf{N} \tag{23}$$

where,

$$\mathbf{W} = [\mathbf{W}_1 \; \mathbf{W}_2 \; \cdots \; \mathbf{W}_R] \tag{24}$$

$$\mathbf{\Gamma} = \text{diag}(\underbrace{\gamma_1, \ldots, \gamma_1}_{K+L-1}, \ldots, \underbrace{\gamma_R, \ldots, \gamma_R}_{K+L-1}) \tag{25}$$

$$\mathbf{S} = [\mathbf{S}_1^T \ \mathbf{S}_2^T \ \cdots \ \mathbf{S}_R^T]^T \quad (26)$$

γ_i $(i = 1, \ldots, R)$ is the channel gain of the i^{th} user. Assume that both symbol and noise are zero-mean i.i.d. random variables with variance σ_s^2 and σ_n^2, respectively. And assume σ_s^2 is known to be 1. The data covariance matrix is therefore

$$\mathbf{R}_x = \frac{1}{N-K+1}\mathrm{E}\{\mathbf{X}\mathbf{X}^H\} = \mathbf{W}\mathbf{\Gamma}\mathbf{\Gamma}^H\mathbf{W}^H + \sigma_n^2\mathbf{I} \quad (27)$$

Hence, a good estimate of the user channel gain is given by

$$\hat{\mathbf{\Gamma}}\hat{\mathbf{\Gamma}}^H = \mathrm{diag}\{\hat{\mathbf{W}}^\dagger(\hat{\mathbf{R}}_x - \hat{\sigma}_n^2\mathbf{I})(\hat{\mathbf{W}}^\dagger)^H\} \quad (28)$$

where, $\dagger$ denotes the left pseudo inverse. The estimate of the noise variance $\hat{\sigma}_n^2$ is obtained as the mean of the $ML_cK - K - L + 1$ smallest eigenvalues of $\hat{\mathbf{R}}_x$. The absolute value of the channel gain can be obtained by a $(K+L-1)$-smoothing on $|\gamma_i|^2$. In order to estimate the unknown phase, we observe that in the noise free case

$$\mathbf{X} = \mathbf{W}\mathbf{\Gamma}\mathbf{S} = \mathbf{W}|\mathbf{\Gamma}|\mathbf{\Phi}\mathbf{S} \quad (29)$$

where, $|\mathbf{\Gamma}| = \mathrm{diag}\{|\gamma_1|, \ldots, |\gamma_1|, \ldots, |\gamma_R|, \ldots, |\gamma_R|\}$, $\mathbf{\Phi} = \mathrm{diag}\{e^{j\phi_1}, \ldots, e^{j\phi_1}, \ldots, e^{j\phi_R}, \ldots, e^{j\phi_R}\}$. Therefore,

$$\mathbf{\Phi}\mathbf{S} = (\mathbf{W}|\mathbf{\Gamma}|)^\dagger\mathbf{X} \quad (30)$$

If the entries in $\mathbf{S}$ are either 1 or -1, the phase can be estimated by squaring the entries of both sides of (30)

$$(\mathbf{\Phi} \odot \mathbf{\Phi})(\mathbf{S} \odot \mathbf{S}) = [(\mathbf{W}|\mathbf{\Gamma}|)^\dagger\mathbf{X}] \odot [(\mathbf{W}|\mathbf{\Gamma}|)^\dagger\mathbf{X}] \quad (31)$$

where, $\odot$ denotes the Hadamard product. $(\mathbf{S} \odot \mathbf{S})$ is a matrix whose entries are all equal to one, and therefore it is easy to solve $e^{j\phi_i}$ in

$$(\mathbf{\Phi} \odot \mathbf{\Phi}) = \mathrm{diag}\{\underbrace{e^{j2\phi_1}, \ldots, e^{j2\phi_1}}_{K+L-1}, \ldots, \underbrace{e^{j2\phi_R}, \ldots, e^{j2\phi_R}}_{K+L-1}\} \quad (32)$$

IV. Simulation

A W-CDMA system is simulated, in which a base station receives signals from $R = 2$ mobile users. The BS has $M = 4$ antennas spaced $\lambda/2$ in a linear array. The carrier frequency is 2 GHz. There are two dominant scattering ray bundles from each user arriving at the BS, with nominal DOAs and nominal delays uniformly distributed on $[0, 2\pi)$ and $[0, LcT]$, respectively. T is the chip period. We assume that the nominal DOAs and delays are pre-estimated. Within each ray bundle, there are 3 individual multipaths, with pass losses and delays uniformly distributed on [0 dB, 20 dB] and $[0, 4T]$, respectively. The maximum angle spread is 5 degrees from the nominal DOA.

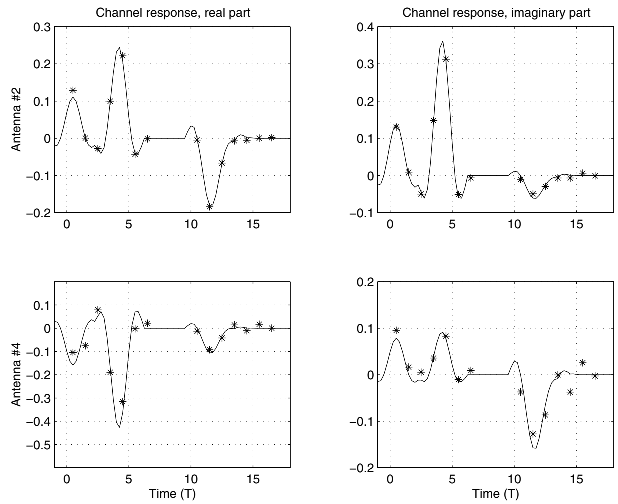

Fig. 1. Channel estimation for user 1. (— : actual channel, * : estimated channel samples.)

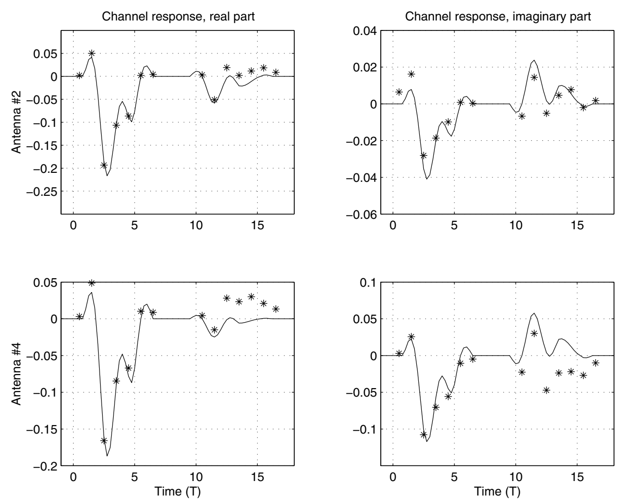

Fig. 2. Channel estimation for user 2. (— : actual channel, * : estimated channel samples.)

We generate random QPSK signals for both mobile users, which are then spread by the the CDMA spreading code of length $L_c = 16$, and modulated by a raised-cosine pulse function with a roll-off factor of 0.25, truncated to a length of $4T$. The average received power of the signal from user 2 is 6 dB lower than that from user 1.

The proposed method of channel estimation is applied with $N = 100$, $K = 2$. L_g is selected to be 7. Once the channel $\bar{\mathbf{h}}_i$ $(i = 1, 2)$ is estimated, we can construct the signature waveform matrix $\mathbf{W}$ and detect the symbols transmitted by each user.

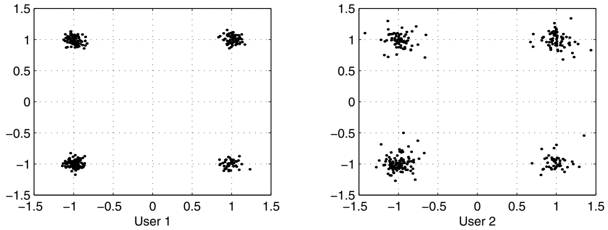

Fig. 3. Signal constellations of user 1 and user 2.

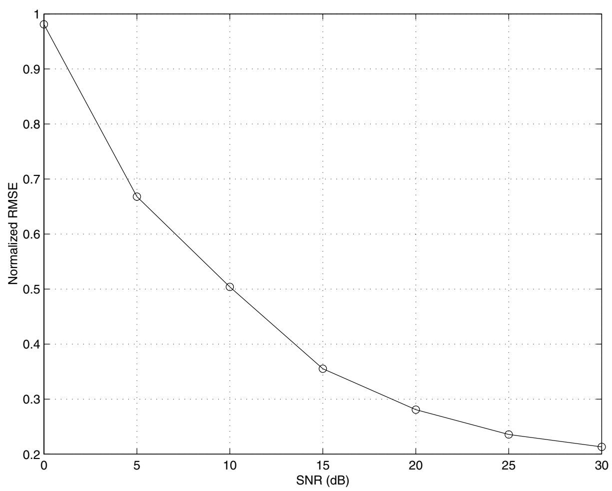

Fig. 4. Normalized RMSE versus channel output SNR.

An example of channel estimation is demonstrated in Fig. 1 and Fig. 2, where SNR = 10 dB. They show the agreement of the actual channel and the estimated channel samples at the second and fourth antenna elements of both users. Although the signals are QPSK encoded, the channel gain and the phase can be estimated by a similar approach as in section III. The compensated signal constellations are shown in Fig. 3.

We define the normalized root mean square error as $\mathrm{RMSE} = \frac{1}{||\bar{\mathbf{h}}||}\sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}||\hat{\bar{\mathbf{h}}}-\bar{\mathbf{h}}||^2}$, where the number of Monte Carlo trials is chosen as $N_t = 1000$. Fig. 4 shows the normalized RMSE of channel estimation with channel output SNR varying from 0 to 30 dB.

In Fig. 5, we compare the average bit-error-rate (BER) for the W-CDMA receiver applying the subspace method with that for the RAKE receiver with pseudo inverse beamforming. The 2-finger RAKE receiver has a finger for each dominant ray bundle, and the 6-finger RAKE receiver has a finger for each individual multipath component. The proposed method outperforms the RAKE receiver at high SNR, because it cancels the interference from the other user by estimating both channels.

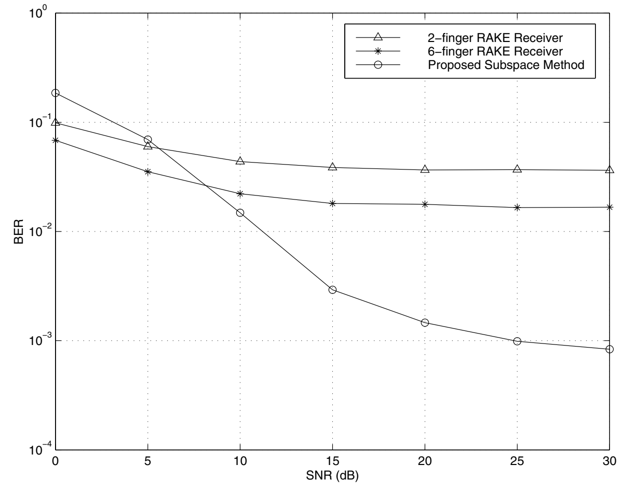

Fig. 5. Average BER versus channel output SNR.

V. Conclusion

We have presented an algorithm for the subspace-based multiuser channel estimation in W-CDMA systems. The algorithm uses information of the slow-varying large-scale structure of the propagation channel and exploits the subspace properties of the received data matrix in a deterministic framework. This approach is computationally more efficient, while preserving the performance of the traditional blind channel estimation techniques.

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