Robust Beamforming for FDD Mobile Systems Over Rayleigh Fading Channels

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***Abstract*— A beamforming approach is developed for mobile wireless systems operated in vehicular scenarios such that the channel undergoes fast Rayleigh fading. In this approach, the downlink beamforming weights of the base antennas are generated based on the orthogonal basis of the uplink signal subspace. The downlink transmission to a particular mobile is through its effective signal subspace via multiple diversity branches. Thanks to the stability of the signal subspace, the received signal strength at the mobile is robust to fast Rayleigh fading. When the distance between the uplink and downlink subspaces is negligible, the open-loop beamforming can be applied to FDD systems. Ray tracing simulations in a three-dimensional urban physical model show that the subspace beamforming enhances the link quality of mobile systems over Rayleigh fading channels.**

I. Introduction

Spatial diversity, obtained through antenna array processing, provides an attractive means to improve the performance of mobile communication systems. When the base transceiver station (BTS) is equipped with an antenna array, and the channels of the array elements to the mobile station (MS) are correlated, the transmitted signals can be properly weighted on each array element to enhance the link quality. In frequency-division duplex (FDD) systems, the direct use of the uplink (mobile to base) antenna weights for downlink (base to mobile) transmission is not advisable due to the uncorrelated fading at the two duplex frequencies and the frequency-dependent antenna array responses [1], [2]. Moreover, most transmission beamforming algorithms adapt only to slowly varying components in the channel and can hardly track the fast fading effect in mobile wireless systems. We refer *fast* Rayleigh fading to the case in which the channel fading rate is comparable to the data frame rate.

In FDD systems, the estimated discrete direction of arrivals (DOAs) of uplink can be used to calculate the antenna weights for downlink transmission [3]. But it is computationally exhaustive and may not be adapted to angular spreading and distributed sources [4], [5]. In case of a large angular spread as in urban environments, the robustness of high-resolution DOA estimation methods is substantially degraded. Beamforming algorithms based on spatial covariance matrices adjust the antenna patterns to the actual propagation conditions and therefore outperform the DOA-based methods [6], [7]. To design the downlink beamformer, the uplink spatial covariance matrix is averaged followed by matrix transformation from uplink frequency to downlink frequency. This is because the mean values of the channel characteristics are the same but the antenna array responses are different at different frequencies [8]–[11]. However, the estimate of the channel spatial covariance matrix averages out the fades therefore the scheme does not allow for reaction on fast fading.

In order to combat fast fading, predictive beamforming [12] can be employed at the BTS that uses the MS feedback of downlink channel estimates [13]. However, when the uplink bandwidth for signaling is limited, the feedback delay makes it difficult to adjust the antenna weights fast enough to account for fast fading. One way to achieve a lower feedback bandwidth is to use an eigenbeamformer [14], [15]. Instead of feeding back the channel estimates for downlink beamforming, only the principle eigenvector of the spatial covariance matrix is determined at the MS and fed back to the BTS. An extension to eigenbeamformer idea is to form transmit antenna weight as a linear combination of the eigenbeams, which include the eigenvector of the long-term spatial covariance matrix estimated at the BTS, and the short-term knowledge of the downlink channel fed back from the MS [16].

In this paper, an adaptive beamforming in the signal subspace is proposed to cope with the issues of FDD links over fast fading channels. As the signal vector varies under fast Rayleigh fading, it is confined to the signal subspace. The user data is transmitted through multiple eigenmodes of the signal subspace. Due to the stable structure of the signal subspace, the mobile can retrieve data that is robust to fast fading. Furthermore, when open-loop beamforming is concerned without MS feedback, the larger the FDD carrier gap, the less correlated are the two principal eigenvectors of the uplink and downlink spatial covariance matrices, thus larger performance loss of the conventional eigenbeamformer. However, the proposed beamforming technique is applicable to an open-loop FDD system when the distance between the uplink and downlink signal subspaces is negligible. The enhanced link quality is acquired by utilizing system diversity resources and by modifying mobile reception.

II. System Model

Consider K active MSs in a wireless communication cell. These K MSs communicate simultaneously with a BTS using a common uplink carrier and a common downlink carrier.

The BTS has an M-element antenna array and each MS has one antenna, hence a vector channel between the BTS and an individual MS. Assuming that the delay spread of the multipath channel is under the chip duration, we apply the narrowband channel model. During uplink, the baseband signal received at the BTS can be expressed as

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}_k^{(u)}(t)\sqrt{p_k^{(u)}}s_k^{(u)}(t) + \mathbf{n}(t) \tag{1}$$

where superscript (u) denotes uplink, and $s_k^{(u)}(t)$ is the signal from MS k with fixed transmit power $p_k^{(u)}$. $\mathbf{n}(t)$ is the receiver background noise vector, each element of which is an independent AWGN with one-sided spectrum density N_0. $\mathbf{a}_k^{(u)}(t)$ is the time-varying spatial signature associated with MS k, which is given by

$$\mathbf{a}_k^{(u)}(t) = \sum_{i=1}^{L_k} \alpha_{k,i}^{(u)}(t)\mathbf{v}^{(u)}(\Omega_{k,i}) \tag{2}$$

where L_k is the total number of paths from MS k. $\alpha_{k,i}(t)$ is the complex amplitude of the i^{th} path from MS k, which is time-varying due to propagation loss, shadowing effect, and multipath fading. $\mathbf{v}^{(u)}(\Omega_{k,i})$ is the steering vector of the M-element antenna array at the uplink frequency associated with direction of arrival (DOA) $\Omega_{k,i} = (\theta_{k,i}, \phi_{k,i})$, where $\theta_{k,i}$ is the azimuth angle, and $\phi_{k,i}$ is the elevation angle. Only azimuthal DOA is considered in this paper for clear demonstration. The steering vector can be written as

$$\mathbf{v}^{(u)}(\theta_{k,i}) = \left[e^{jk^{(u)}d_1(\theta_{k,i})}, \cdots, e^{jk^{(u)}d_M(\theta_{k,i})}\right]^T \tag{3}$$

where $d_m(\theta)$ is the relative distance of the m^{th} antenna element along DOA θ, and $k^{(u)}$ is the uplink wave number as $k^{(u)} = \frac{2\pi f_c^{(u)}}{C}$. $f_c^{(u)}$ is the uplink carrier frequency, and C is the wave propagation speed. The transmitted signal $s_k(t)$ depends on the information-bearing data stream $\{b_k^{(u)}(n)\}$ and the modulation waveform $g_k(t)$ as

$$s_k^{(u)}(t) = \sum_n b_k^{(u)}(n)g_k(t - nT_s) \tag{4}$$

where T_s is the symbol period. The variation of the spatial signature $\mathbf{a}_k^{(u)}(t)$ over a symbol period T_s is assumed negligible. Suppose a matched filter $g_k^*(-t)$ with perfect synchronization is present at the receiver, then the output of the matched filter for the signal from MS k sampled at the symbol rate is

$$\mathbf{y}_k(n) = \sqrt{Gp_k^{(u)}}\mathbf{a}_k^{(u)}(n)b_k^{(u)}(n) + \tilde{\mathbf{n}}_k(n) \tag{5}$$

where G is the processing gain, $\tilde{\mathbf{n}}_k(n)$ is the undesired component consisting of interference and noise.

The BTS adjusts its antenna array beam pattern by assigning a weight vector $\mathbf{w}_k$, such that the array output is weighted and added to maximize the signal-to-interference-and-noise-ratio (SINR). The average output power of the received symbols is

$$\begin{aligned}\mathcal{P} &= \mathrm{E}\{\mathbf{w}_k^H\mathbf{y}_k(n)\mathbf{y}_k^H(n)\mathbf{w}_k\} \\ &= \mathbf{w}_k^H\mathrm{E}\{\mathbf{y}_k(n)\mathbf{y}_k^H(n)\}\mathbf{w}_k \\ &= \mathbf{w}_k^H\mathbf{R}_{yy}\mathbf{w}_k \end{aligned} \tag{6}$$

where ^{H} denotes conjugate transpose, $\mathbf{R}_{yy}$ is the spatial covariance matrix of the matched filter output. If the channel varies slowly and the spatial signatures are invariant during uplink, $\mathbf{R}_{yy}$ can be written as

$$\mathbf{R}_{yy} = Gp_k\mathbf{a}_{k,i}\mathbf{a}_{k,i}^H + \mathbf{R}_{in} \tag{7}$$

where $\mathbf{R}_{in}$ is the spatial covariance matrix of total interference plus noise. The weight vector which maximizes the average SINR

$$\Gamma = \frac{Gp_k\mathbf{w}_{k,i}^H\mathbf{a}_{k,i}\mathbf{a}_{k,i}^H\mathbf{w}_{k,i}}{\mathbf{w}_{k,i}^H\mathbf{R}_{in}\mathbf{w}_{k,i}} \tag{8}$$

is given by the optimum Wiener solution as [17]

$$\mathbf{w}_{k,i} = \zeta\mathbf{R}_{in}^{-1}\mathbf{a}_{k,i} \tag{9}$$

where ζ is an arbitrary constant that takes into account the power constraint. Assuming that the interference is spatially white, $\mathbf{R}_{in}$ is proportional to an identity matrix. Therefore, a suboptimal solution for the weight vector is given by

$$\mathbf{w}_{k,i} = \xi\mathbf{a}_{k,i} \tag{10}$$

The optimum combining and suboptimum combining have similar performance [18]. Assume that the modulation waveforms of the interfering users appear as mutually uncorrelated noise. Therefore, the desired signal is sufficiently stronger than interference and noise after matched filtering, and $\mathbf{a}_{k,i}$ can be obtained as the principal eigenvector of $\mathbf{R}_{yy}$.

III. Robust Beamforming

Under fast fading, the spatial signatures $\{\mathbf{a}_k(t)\}$ can vary drastically during a transmission period. The signal received at the MSs may exhibit deep fades if the BTS uses conventional beamforming methods. Moreover, in FDD systems, the instantaneous channels for uplink and downlink are uncorrelated when the carrier frequency separation is larger than the channel coherence bandwidth [19]. Therefore the uplink spatial signatures $\{\mathbf{a}_k^{(u)}(t)\}$ can not be used directly for downlink transmission. In order to combat fast fading, the BTS transmits the signal through multiple eigenmodes of the signal subspace. The received power of the desired signal at the MS is preserved over a fast fading channel. This transmission method exploits the signal subspaces instead of the instantaneous spatial signatures and is applicable to practical FDD systems without channel feedback requirement.

A. Fading-Resistant Beamforming

As revealed in (2), the uplink spatial signature $\mathbf{a}^{(u)}(t)$ is a linear combination of the uplink steering vectors $\{\mathbf{v}^{(u)}(\theta_i)\}$. The same applies to the downlink spatial signature $\mathbf{a}^{(d)}(t)$. Here, the subscript k is omitted with the understanding that MS k is under consideration. Therefore,

$$\begin{aligned}\mathbf{a}^{(u)}(t) &\in \mathcal{A}^{(u)} \\ &= \text{span}\left\{\mathbf{v}^{(u)}(\theta_1), \mathbf{v}^{(u)}(\theta_2), \ldots, \mathbf{v}^{(u)}(\theta_{L_k})\right\}\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{a}^{(d)}(t) &\in \mathcal{A}^{(d)} \\ &= \text{span}\left\{\mathbf{v}^{(d)}(\theta_1), \mathbf{v}^{(d)}(\theta_2), \ldots, \mathbf{v}^{(d)}(\theta_{L_k})\right\}\end{aligned} \tag{12}$$

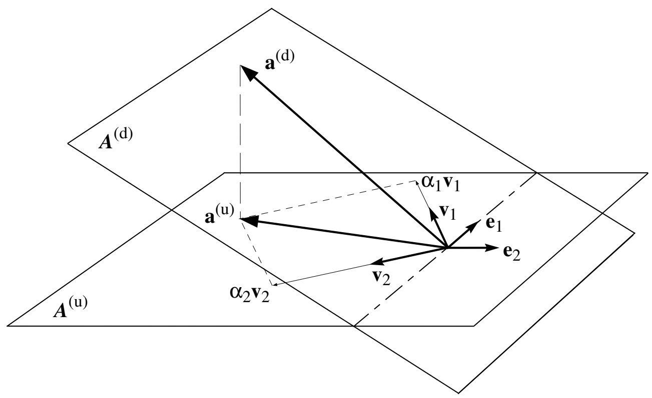

Fig. 1. Uplink and downlink signal subspaces.

where $\mathcal{A}^{(u)}$ and $\mathcal{A}^{(d)}$ are defined as the uplink signal subspace and the downlink signal subspace, respectively. The DOAs $\{\theta_i\}$ are large-scale channel parameters and assumed invariant over the time of interest, therefore the bases $\{\mathbf{v}^{(u)}(\theta_i)\}$ and $\{\mathbf{v}^{(d)}(\theta_i)\}$ hence the signal subspaces $\mathcal{A}^{(u)}$ and $\mathcal{A}^{(d)}$ are fixed. The rapid time-variance of the spatial signatures $\mathbf{a}^{(u)}(t)$ and $\mathbf{a}^{(d)}(t)$ is primarily due to multipath fading. Nevertheless, they are confined in the signal subspaces as shown in Fig. 1. Assuming L paths have predominant gains among a total of L_k paths, and the steering vectors $\{\mathbf{v}^{(u)}(\theta_i)\}$ and $\{\mathbf{v}^{(d)}(\theta_i)\}$ of these paths are two sets of linearly independent vectors, (therefore $L \leq M$), we define L as the effective rank of $\mathcal{A}^{(u)}$ and $\mathcal{A}^{(d)}$ as

$$L = \mathcal{R}(\mathcal{A}^{(u)}) = \mathcal{R}(\mathcal{A}^{(d)}) \tag{13}$$

Without loss of generality, indexes 1 to L are assigned to these predominant paths.

From (2) and (5), the uplink signal at the output of the BTS matched filter can be written as

$$\mathbf{y}(n) \approx \sqrt{Gp^{(u)}}b^{(u)}(n)\sum_{i=1}^{L}\alpha_i^{(u)}(n)\mathbf{v}^{(u)}(\theta_i) + \tilde{\mathbf{n}}(n) \tag{14}$$

The Rayleigh fading is due to the independent phase changes of $\{\alpha_i^{(u)}(n)\}$ caused by the Doppler effect. Suppose $\mathbf{y}(n)$ is wide-sense stationary, and the path gains $\{|\alpha_i^{(u)}|^2\}$ do not change over time, the spatial sample covariance matrix of the uplink signal can be obtained by

$$\begin{aligned}\hat{\mathbf{R}}_{yy} &= \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}(n)\mathbf{y}^H(n) \\ &= Gp^{(u)}\mathbf{V}\mathbf{\Lambda}\mathbf{V}^H + \hat{\mathbf{R}}_{in}\end{aligned} \tag{15}$$

where

$$\mathbf{V} = \left[\mathbf{v}^{(u)}(\theta_1)\ \mathbf{v}^{(u)}(\theta_2)\ \cdots\ \mathbf{v}^{(u)}(\theta_L)\right] \tag{16}$$

$$\mathbf{\Lambda} = \text{diag}\left\{|\alpha_1^{(u)}|^2, |\alpha_2^{(u)}|^2, \ldots, |\alpha_L^{(u)}|^2\right\} \tag{17}$$

and $\hat{\mathbf{R}}_{in}$ is the interference plus noise component. We assume that the paths of different DOAs are uncorrelated, and the interference and noise are uncorrelated between the antennas. As $N \to \infty$, $\hat{\mathbf{R}}_{in} \to \sigma_w^2\mathbf{I}$, where σ_w^2 is the variance of interference plus noise at each antenna. The L principle eigenvectors of $\mathbf{R}_{yy}$, corresponding to the L largest eigenvalues, form an (effective) orthonormal basis $\{\mathbf{e}_i, i = 1, \ldots, L\}$ of the signal subspace.

Let us first consider the case where these is no carrier frequency separation between uplink and downlink, e.g. in a time-division duplex (TDD) system. In such case, the steering vectors hence the signal subspaces are identical, therefore $\mathcal{A}^{(u)}$ and $\mathcal{A}^{(d)}$ coincide in Fig. 1. The orthonormal basis $\{\mathbf{e}_i\}$ obtained based on the uplink signal is an effective basis of the downlink subspace, thus the downlink spatial signature $\mathbf{a}^{(d)}(t)$ approximates a linear combination as

$$\mathbf{a}^{(d)}(t) \approx \sum_{i=1}^{L}\mathbf{e}_i^H\mathbf{a}^{(d)}(t)\mathbf{e}_i = \sum_{i=1}^{L}\beta_i(t)\mathbf{e}_i \tag{18}$$

where $\beta_i(t) = \mathbf{e}_i^H\mathbf{a}^{(d)}(t)$ is the projection of the downlink spatial signature onto the basis vector $\mathbf{e}_i$ of the uplink signal subspace. We have

$$\sum_{i=1}^{L}|\beta_i(t)|^2 \approx ||\mathbf{a}^{(d)}(t)||^2 \tag{19}$$

The BTS transmits L replicas of data to MS k, and assigns the orthonormal basis vector $\mathbf{e}_i$ as the antenna weight for each branch. The transmitted signal intended for MS k is given by a row vector

$$\mathbf{s}^{(d)}(t) = \sum_{i=1}^{L}\mathbf{e}_i^H s_i^{(d)}(t) \tag{20}$$

The signal received by MS k is

$$\begin{aligned}x(t) &= \mathbf{s}^{(d)}(t)\mathbf{a}^{(d)}(t) + n(t) \\ &= \sum_{i=1}^{L}\mathbf{e}_i^H\mathbf{a}^{(d)}(t)s_i^{(d)}(t) + n(t)\end{aligned} \tag{21}$$

In order to recover each replica of the transmitted signal at the receiver, the L branches are independently modulated. This can be implemented by combining beamforming and transmit diversity [20]. Suppose that each branch bears the same downlink information symbols $\{b^{(d)}(n)\}$ and can be extracted as

$$\begin{aligned}x_i(n) &= \mathbf{e}_i^H\mathbf{a}^{(d)}(n)b^{(d)}(n) + \eta_i(n) \\ &= \beta_i(n)b^{(d)}(n) + \eta_i(n), \quad i = 1, \ldots, L\end{aligned} \tag{22}$$

where $\eta_i(n)$ contains the interference from other branches of the intended user, the multiple access interference, and Gaussian noise. The received signal power of each branch fluctuates within the downlink period due to fast Rayleigh fading, whereas, the total signal power in the signal subspace remains almost constant. At MS k, the extracted signals of the L branches can be combined after phase compensation, for example, using the differential phase shift keying (DPSK) modulation scheme as

$$
\begin{aligned}
\hat{x}(n) &= \sum_{i=1}^{L} x_i^*(n-1)x_i(n) \\
&= \sum_{i=1}^{L} \beta_i^*(n-1)\beta_i(n)b^{(d)*}(n-1)b^{(d)}(n) + w(n) \\
&= \sum_{i=1}^{L} |\beta_i(n)|^2 d^{(d)}(n) + w(n) \\
&\approx ||\mathbf{a}^{(d)}(n)||^2 d^{(d)}(n) + w(n) \qquad (23)
\end{aligned}
$$

where * denotes complex conjugate, $\{d^{(d)}(n)\}$ are the original symbols, $\{b^{(d)}(n)\}$ are the differentially encoded symbols, and $w(n)$ is the interference plus noise component. The downlink spatial signature is assumed to be fixed over a few symbol period, such that $\beta_i(n) = \beta_i(n-1)$. Finally, the original information-bearing bit is detected from $\hat{x}(n)$. Since the signal strength is proportional to $||\mathbf{a}^{(d)}(n)||^2$, the fading is greatly alleviated because it is quite unlikely that all vector components vanish simultaneously.

B. Open-Loop Beamforming for FDD

In FDD systems, reciprocity can be assumed between the uplink and downlink that comprises the DOAs, the path delays, and the average path losses. Experimental measurements at 900 MHz have shown that the DOAs remain relatively stable over the frequency range used for uplink and downlink transmission in GSM [1]. However, the reciprocity does not hold for the phases, thus the uplink and downlink spatial signatures $\mathbf{a}^{(u)}(t)$ and $\mathbf{a}^{(d)}(t)$ vary independently. In spite of the independent fading of uplink and downlink, the subspace beamforming proposed is an open-loop scheme such that the feedback of the knowledge of the downlink spatial signature is not necessary.

The BTS assigns L orthonormal basis vectors of the uplink signal subspace as weights for downlink transmission. As the carrier frequency differs in FDD, the uplink steering vector $\mathbf{v}^{(u)}(\theta_i)$ is different from the downlink steering vector $\mathbf{v}^{(d)}(\theta_i)$. Therefore, the signal subspaces $\mathcal{A}^{(u)}$ and $\mathcal{A}^{(d)}$ do not coincide. Similar to (23), the combination of the received signals at MS k is given by

$$
\begin{aligned}
\hat{x}(n) &= \sum_{i=1}^{L} |\beta_i(n)|^2 d^{(d)}(n) + w(n) \\
&\approx ||P^{(u)}\mathbf{a}^{(d)}(n)||^2 d^{(d)}(n) + w(n) \qquad (24)
\end{aligned}
$$

where $P^{(u)} \in \mathbb{C}^{M\times M}$ is the orthogonal projection onto the uplink signal subspace $\mathcal{A}^{(u)}$. Let $P^{(d)} \in \mathbb{C}^{M\times M}$ denote the orthogonal projection onto the downlink signal subspace $\mathcal{A}^{(d)}$. Omitting the time index n, we have

$$
\begin{aligned}
||P^{(u)}\mathbf{a}^{(d)}||^2 &= ||\mathbf{a}^{(d)}||^2 - ||(I - P^{(u)})\mathbf{a}^{(d)}||^2 \\
&= ||\mathbf{a}^{(d)}||^2 - ||(P^{(d)} - P^{(u)})\mathbf{a}^{(d)}||^2 \\
&\geq ||\mathbf{a}^{(d)}||^2 - ||P^{(d)} - P^{(u)}||_2^2 \cdot ||\mathbf{a}^{(d)}||^2 \\
&= (1 - \mathcal{D}^2(\mathcal{A}^{(u)}, \mathcal{A}^{(d)}))||\mathbf{a}^{(d)}||^2 \qquad (25)
\end{aligned}
$$

Suppose the uplink and downlink signal subspaces $\mathcal{A}^{(u)}$ and $\mathcal{A}^{(d)}$ have the same dimension, the distance between the signal subspaces can be defined by [21]

$$\mathcal{D}(\mathcal{A}^{(u)}, \mathcal{A}^{(d)}) = ||P^{(d)} - P^{(u)}||_2 \qquad (26)$$

where $||\cdot||_2$ denotes matrix 2-norm. The orthonormal bases of $\mathcal{A}_{(u)}$ and $\mathcal{A}_{(d)}$ can be expressed as

$$
\begin{aligned}
\mathbf{U}^{(u)} &= \left[\mathbf{e}_1^{(u)}\mathbf{e}_2^{(u)}\cdots\mathbf{e}_{L'}^{(u)}\right] \qquad (27) \\
\mathbf{U}^{(d)} &= \left[\mathbf{e}_1^{(d)}\mathbf{e}_2^{(d)}\cdots\mathbf{e}_{L'}^{(d)}\right] \qquad (28)
\end{aligned}
$$

where L' is the rank of the uplink and downlink signal subspaces, such that $L \leq L' \leq M$. The distance between the signal subspaces is given by [21]

$$\mathcal{D}(\mathcal{A}^{(u)}, \mathcal{A}^{(d)}) = \sqrt{1-\sigma^2} \qquad (29)$$

where $\sigma \in (0,1)$ is the smallest singular value of $\mathbf{U}^{(u)H}\mathbf{U}^{(d)}$. Substituting the subspace distance in (25) with (29), we have

$$||P^{(u)}\mathbf{a}^{(d)}|| \geq \sigma||\mathbf{a}^{(d)}|| \qquad (30)$$

The FDD reception is ameliorated if the signal strength $||P^{(u)}\mathbf{a}^{(d)}||^2$ does not deteriorate over time. This requires a small distance between the signal subspaces such that σ is close to 1. As shown in Section IV-C, this is easy to design and is valid in most practical FDD systems.

IV. Performance Metrics

A. Effective Rank of Signal Subspace

The effective rank L of the signal subspace can be determined, along the lines of [22], based on the application of the information theoretic criteria for model selection [23], [24]. It can be determined as the number of dominant eigenvalues of $\mathbf{R}_{yy}$. Assume that L' is the rank of the signal subspace, L is the effective rank of the signal subspace as defined in Section III-A, and $L \leq L' \leq M$. The projection of the downlink spatial signature can be expressed as

$$
\begin{aligned}
P^{(u)}\mathbf{a}^{(d)}(n) &= \sum_{i=1}^{L'} \mathbf{e}_i^H\mathbf{a}^{(d)}(n)\mathbf{e}_i \\
&= \sum_{i=1}^{L} \beta_i(n)\mathbf{e}_i + \sum_{i=L}^{L'} \beta_i(n)\mathbf{e}_i \qquad (31)
\end{aligned}
$$

Therefore, the combined signal strength at the MS receiver is proportional to

$$\sum_{i=1}^{L} |\beta_i(n)|^2 = ||P^{(u)}\mathbf{a}^{(d)}(n)||^2 - \sum_{i=L}^{L'} |\beta_i(n)|^2 \qquad (32)$$

When an eigenvector $\mathbf{e}_m$ corresponding to an eigenvalue λ_m is used as the beamforming weight for one signal branch, the average branch contribution to the overall signal strength is

$$
\begin{aligned}
\mathrm{E}\{|\beta_m(n)|^2\} &= \mathbf{e}_m^H \mathrm{E}\{\mathbf{a}^{(d)}(n)\mathbf{a}^{(d)H}(n)\}\mathbf{e}_m \\
&\approx \frac{1}{Gp^{(u)}}\mathbf{e}_m^H(\mathbf{R}_{yy} - \sigma_w^2\mathbf{I})\mathbf{e}_m \\
&= \frac{1}{Gp^{(u)}}(\lambda_m - \sigma_w^2) \qquad (33)
\end{aligned}
$$

where σ_w^2 is the variance of interference plus noise at each antenna, and $\sigma_w^2 \leq \lambda_m$. The second step in (33) makes the assumption that the downlink spatial covariance matrix approximates the uplink spatial covariance matrix. The smaller the eigenvalue λ_m, the less average contribution is this branch to the overall signal strength. Therefore, excluding the last $L'-L$ terms corresponding to the $L'-L$ smallest eigenvalues does not statistically affect the claim that

$$||P^{(u)}\mathbf{a}^{(d)}(n)||^2 \approx \sum_{i=1}^{L} |\beta_i(n)|^2 \tag{34}$$

B. Channel Norm

As in (24), the combined signal strength is proportional to $||P^{(u)}\mathbf{a}^{(d)}(n)||^2$. From (30), we have $||P^{(u)}\mathbf{a}^{(d)}(n)||^2 \geq \sigma^2||\mathbf{a}^{(d)}(n)||^2$. For a spatially white channel, $||\mathbf{a}^{(d)}(n)||^2$ has a χ_M^2 distribution. The amplitude $||\mathbf{a}^{(d)}(n)||$ varies slowly compared to the vector angle of the spatial signature. Over fast Rayleigh fading channels, the conventional eigenbeamformer will result in deep fades in the received signal due to rapid angle variation. The proposed beamforming algorithm circumvents this problem by attributing the received signal strength to the more tractable vector amplitude of the fading channel. Suppose that the downlink steering vector is given similar to (3), and the complex amplitude of the p^{th} downlink path can be written as

$$\alpha_p^{(d)}(n) = \rho_p e^{j2\pi(f_p nT_s-(f_c+f_p)\tau_p)} \tag{35}$$

where f_c is the downlink carrier frequency, ρ_p, f_p, and τ_p are the complex gain, Doppler frequency, and path delay of the p^{th} path, respectively. We have

$$\begin{aligned} ||\mathbf{a}^{(d)}(n)||^2 &= \mathbf{a}^{(d)H}(n)\mathbf{a}^{(d)}(n) \\ &= \sum_p\sum_q \alpha_p^{(d)*}(n)\alpha_q^{(d)}(n)\mathbf{v}^{(d)H}(\theta_p)\mathbf{v}^{(d)}(\theta_q) \\ &= M\sum_p|\rho_p|^2 \\ &\quad +2\Re e\left\{\sum_{p<q}\gamma_{p,q}e^{j2\pi(f_q-f_p)nT_s}\right\} \end{aligned} \tag{36}$$

where $\Re e\{\cdot\}$ denotes the real part of a complex value, and $\gamma_{p,q} = \rho_p^*\rho_q\mathbf{v}_p^H\mathbf{v}_q e^{j2\pi(f_p\tau_p-f_q\tau_q+f_c(\tau_p-\tau_q))}$. $|\mathbf{v}_p^H\mathbf{v}_q| < ||\mathbf{v}_p||^2 = M$ when $p \neq q$. Note that, in (36), $||\mathbf{a}^{(d)}(n)||^2$ has a large positive term as the first sum, and each component of the second sum has independent phase change. Therefore, the second sum can be approximated by a zero-mean Gaussian random variable with variance less than $M\sum_p|\rho_p|^2$. This implies that a large negative value of the second sum with respect to the first sum is quite unlikely.

C. Distance between FDD Subspaces

To design a BTS antenna array for FDD systems, we employ array aperture to make the uplink- and downlink subspaces close. The distance between two subspaces can be indicated by the value of σ as in (29). We know that $\sigma = \sigma_{\min}\{\mathbf{\Psi}\}$, where $\sigma_{\min}\{\cdot\}$ denotes the smallest singular value, and

$$\begin{aligned} \mathbf{\Psi} &= \mathbf{U}^{(u)H}\mathbf{U}^{(d)} \\ &= \begin{bmatrix} \mathbf{e}_1^{(u)H}\mathbf{e}_1^{(d)} & \mathbf{e}_1^{(u)H}\mathbf{e}_2^{(d)} & \cdots & \mathbf{e}_1^{(u)H}\mathbf{e}_{L'}^{(d)} \\ \vdots & \vdots & & \vdots \\ \mathbf{e}_{L'}^{(u)H}\mathbf{e}_1^{(d)} & \mathbf{e}_{L'}^{(u)H}\mathbf{e}_2^{(d)} & \cdots & \mathbf{e}_{L'}^{(u)H}\mathbf{e}_{L'}^{(d)} \end{bmatrix} \end{aligned} \tag{37}$$

Suppose that L' linearly independent uplink steering vectors $\{\mathbf{v}^{(u)}(\theta_i)\}$ and L' linearly independent downlink steering vectors $\{\mathbf{v}^{(d)}(\theta_i)\}$ span the uplink and downlink subspaces, respectively. The orthonormal bases $\{\mathbf{e}_i^{(u)}\}$ and $\{\mathbf{e}_i^{(d)}\}$ can be constructed from $\{\mathbf{v}^{(u)}(\theta_i)\}$ and $\{\mathbf{v}^{(d)}(\theta_i)\}$ by the Gram-Schmidt orthogonalization process. In the simulations, we use a uniform circular array to guarantee the small distance between uplink- and downlink subspaces. The steering vectors of a circular array are given by (the origin of the circle being the reference)

$$\begin{aligned} \mathbf{v}^{(u)}(\theta_i) &= \Big[e^{jk^{(u)}R\cos(\theta_i)}, e^{jk^{(u)}R\cos(\theta_i+2\pi/M)}, \\ &\quad \cdots, e^{jk^{(u)}R\cos(\theta_i+2\pi(M-1)/M)}\Big]^T \end{aligned} \tag{38}$$

$$\begin{aligned} \mathbf{v}^{(d)}(\theta_i) &= \Big[e^{jk^{(d)}R\cos(\theta_i)}, e^{jk^{(d)}R\cos(\theta_i+2\pi/M)}, \\ &\quad \cdots, e^{jk^{(d)}R\cos(\theta_i+2\pi(M-1)/M)}\Big]^T \end{aligned} \tag{39}$$

where $i = 1,\ldots,L'$, R is the radius of the circular array, and $k^{(u)}$ and $k^{(d)}$ are the uplink and downlink wave numbers, respectively. Let R equal to half the medium carrier wavelength of uplink and downlink in a FDD system, i.e. $R = \frac{C}{4}\left(\frac{1}{f_c^{(u)}}+\frac{1}{f_c^{(d)}}\right)$. It follows that $k^{(u)}R = \frac{\pi}{2}\left(1+\frac{f_c^{(u)}}{f_c^{(d)}}\right)$ and $k^{(d)}R = \frac{\pi}{2}\left(1+\frac{f_c^{(d)}}{f_c^{(u)}}\right)$. We have

$$\mathbf{v}^{(d)}(\theta_i) = \mathbf{D}_i\mathbf{v}^{(u)}(\theta_i) \tag{40}$$

where,

$$\begin{aligned} \mathbf{D}_i &= \operatorname{diag}\Bigg\{e^{j\frac{\pi}{2}(\frac{f_c^{(d)}}{f_c^{(u)}}-\frac{f_c^{(u)}}{f_c^{(d)}})\cos(\theta_i)}, \\ &\quad \cdots, e^{j\frac{\pi}{2}(\frac{f_c^{(d)}}{f_c^{(u)}}-\frac{f_c^{(u)}}{f_c^{(d)}})\cos(\theta_i+2\pi(M-1)/M)}\Bigg\} \end{aligned} \tag{41}$$

In a practical FDD system, the ratio of carrier separation to carrier frequency is small. Therefore, $\frac{f_c^{(d)}}{f_c^{(u)}} - \frac{f_c^{(u)}}{f_c^{(d)}} \approx 0$, and $\mathbf{D}_i$ is close to an identity matrix. Consequently, $\mathbf{v}^{(u)}(\theta_i)$ and $\mathbf{v}^{(d)}(\theta_i)$, therefore $\mathbf{e}_i^{(u)}$ and $\mathbf{e}_i^{(d)}$, are close in the M-dimensional vector space. The smallest singular value of $\mathbf{\Psi}$ is the length of the shortest semi-axis of the hyperellipsoid $\mathcal{E}$ defined by $\mathcal{E} = \{\mathbf{\Psi}\mathbf{x} : ||\mathbf{x}|| = 1\}$. Because the matrix $\mathbf{\Psi}$ has predominant diagonal entries with almost equal magnitude, $\mathcal{E}$ is close to a hypersphere, thus $\sigma = \sigma_{\min}\{\mathbf{\Psi}\} \approx 1$. In the simulations, the uplink frequency $f_c^{(u)} = 1.8$ GHz and the downlink frequency $f_c^{(d)} = 2.0$ GHz, and the BTS uses a circular array with $M = 8$ antenna elements. In this FDD system with arbitrary DOAs $\{\theta_i\}$, the minimum σ is $\sigma \approx 0.940$ for $L' = 2$, and $\sigma \approx 0.929$ for $L' = 3$. It should be noted that the array used in the simulations is rather small

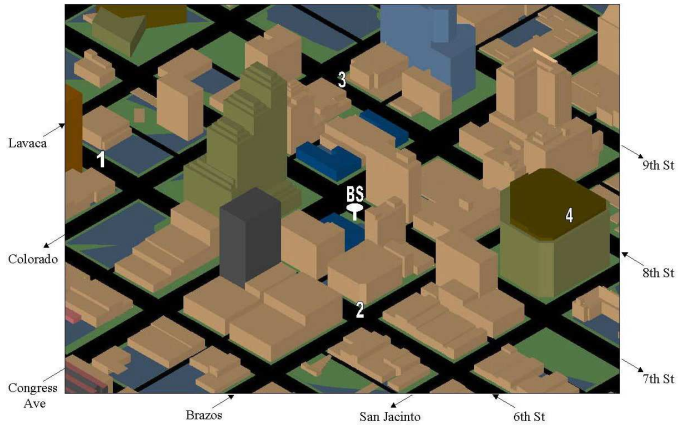

Fig. 2. A mobile communications system which consists of a base transceiver station and four mobile stations in the urban vehicular environment of downtown Austin, Texas. (MS 4 indicated here is behind the building on 8th Street.)

compared to the wavelength. For larger arrays, there may be some performance degradation due to the increased distance between the uplink and downlink subspaces.

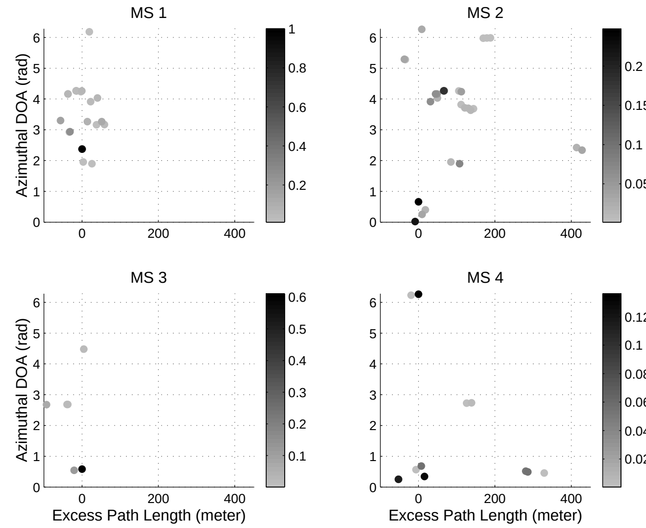

Fig. 3. Azimuthal arriving angles versus excess path lengths of paths viewed at the base transceiver station. Gray bar indicates the relative strength of each path.

V. Simulations

In this section, we give an example of a mobile communications system to compare the performance of the proposed beamforming scheme with the conventional eigenbeamformer. An electromagnetic solver FASANT [25] is used to simulate the propagation environment and the antennas. It is a deterministic ray tracing technique based on geometric optics and the uniform theory of diffraction. A geometry model of downtown Austin, Texas is used as the propagation environment, which has appropriate material properties of the building walls and the ground (Fig. 2). It represents a typical urban center with some high-rise (up to about 30 stories) and low-rise buildings. Fig. 2 shows that the BTS communicates with four randomly positioned MSs in the streets. No mobile has light-of-sight with the BTS antenna array. The BTS antenna array is an 8-element uniform circular array at a height of 20 meter, with a radius of 0.085 meter about half the carrier wavelength. Each array element is a vertically placed omnidirectional dipole antenna. Each mobile has one vertically placed omnidirectional dipole antenna 1.5 m off the ground. The ray tracing outputs of the dominant (in terms of receiving power) paths are shown in Fig. 3. The DOA, delay and field strength are indicated for each path arriving at the BTS. The path delays are expressed as the excess path lengths in meters. We assume that the strongest resolvable paths of the four MSs are synchronized at the BTS.

The communication is FDD with an uplink carrier of 1.8 GHz and a downlink carrier of 2.0 GHz. For comparison, we also simulate a TDD system with a carrier frequency 1.8 GHz for both uplink and downlink. Uplink transmit power control is employed to compensate the near-far effect. The receive frame and the transmit frame have equal duration of 5 ms, each consists of 320 symbols. The symbols are DQPSK modulated, spread with CDMA Walsh codes of length 8, and scrambled with a pseudorandom long code. Therefore the chip rate is 512 KHz, and the propagation length over one chip period is approximately 586 meters. The delay spreads of all the MSs shown in Fig. 3 are within a chip period. This gives flat-fading channels. The pulse-shaping filters at the transmitter and the receiver are root raised cosines with a roll-off factor 0.35.

Fig. 4 compares the eigenvalues of the normalized spatial covariance matrices of the despreaded uplink signals. At a channel SNR of 0 dB, there are two dominant eigenvalues of the spatial covariance matrix of MS 2, and one dominant eigenvalue of each of those of MS 1, 3 and 4. The rest of the eigenvalues are more than 20 dB down the largest ones. MS 2 indeed experiences most severe Rayleigh fading. The BTS assigns multiple code channels to transmit data to MS 2, and the reception performances at MS 2 are evaluated.

Fig. 5 depicts the bit error rate (BER) of downlink reception of MS 2 in the multiuser interfering CDMA. In this case, all MSs move at 30 mph. $L = 2$ and $L = 3$ denote the number of branches used in the subspace beamforming, whereas $L = 1$ denotes the conventional eigenbeamforming, where the transmit weight is constructed as the principle eigenvector of the spatial sample covariance matrix. The BER of the subspace beamforming scheme reduced over an order of magnitude at a channel SNR of 0 dB in both FDD and TDD. As implied in Fig. 4, the effective rank of the signal subspace of MS 2 is two. Consequently, the performance improvement of using three code channels instead of two code channels is not as much as using two code channels instead of conventional eigenbeamforming. This is more clearly demonstrated in TDD. If there are two predominant eigenvalues present, transmission on three eigenmodes in case of TDD can not benefit the reception. But for the FDD system, the effect is shown, that one needs more eigenmodes due to the mismatch of the signal subspaces at uplink and downlink carrier frequencies. The BER curves are lower-bounded by the BER in the scenario where all MSs are stationary.

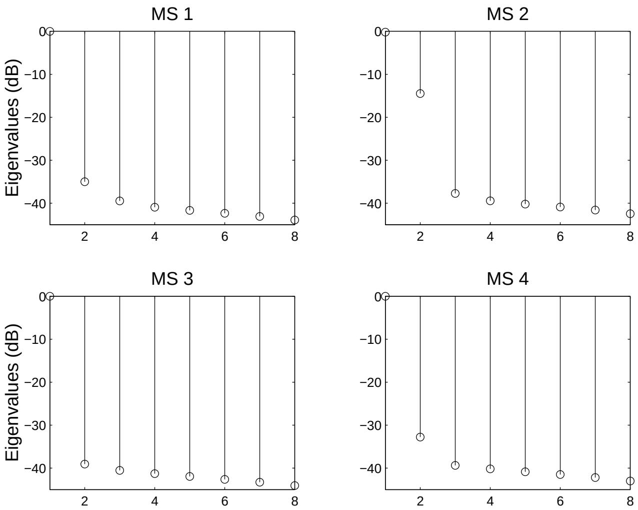

Fig. 4. Eigenvalues of the normalized spatial covariance matrices of the despreaded uplink signals of the 4 MSs. Channel SNR = 0 dB.

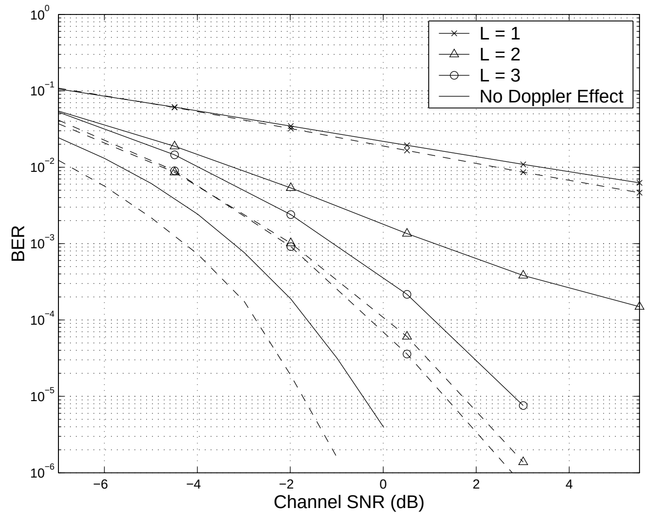

Fig. 5. Bit error rate of DQPSK over a fading channel with AWGN. Mobile speed 30 mph. FDD: solid line, TDD: dashed line.

VI. Conclusion

A beamforming scheme was proposed for the mobile wireless systems to combat fast Rayleigh fading. The algorithm exploits the stability of signal subspaces for communications over fading channels. It is applicable to open-loop FDD systems that have negligible distance between the uplink and downlink signal subspaces. Simulation results on a ray-tracing model show performance improvement of the subspace beamforming scheme over conventional beamforming methods. The superior link performance is acquired by utilizing system diversity resources and by modifying mobile reception.

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