NAME :

Math 570
EXAM 2

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1. Consider the sequence

$$x_n = \frac{(-1)^n n^2}{1+n^2}$$

(a) Is this sequence Cauchy? Explain.

(b) Find limit superior and limit inferior of this sequence.

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2. Let $x_1 = 1$ and $x_{n+1} = \sqrt{1+x_n^2}$ for $n \geq 1$. Show that this sequence is monotone increasing but not converging. Explain.

3. Consider function $f(x) = \frac{1}{x^2}$.
(a) Is this function uniformly continuous on $[1, +\infty)$? on $(0, 1]$? Explain.

(b) Find $\sup f$ and $\inf f$ on the set $S = [1, 10]$. Are maximum or minimum of this function are attained on S? Explain.

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4. Find limits
(a) $\lim\limits_{x \to 0} (1 + x^2)^{\frac{2}{x}} =$

(b) $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{\sin(6x)} =$

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5. Let

$$f(x) = \begin{cases} x^3 \cos(1/x), & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

Find the derivative of this function at any point x and show that it is continuous. (*Hint: to find the derivative of f at $x = 0$ use the definition of the derivative.*)