Math 6050
EXAM 1 (Sample exam)

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1. For each value of α find all stationary points of the function

$$f(x, y) = x^2 + \alpha y^2 - xy + 2x + y$$

Which of these stationary points are global minima?

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2. Find all stationary points of the function

$$f(x, y) = (x^2 - y) + (y - 1)^2$$

and use second-order conditions for optimality to determine all local minima and maxima.

3. Describe Steepest Descent method with Armijo rule for the stepsize choice. Describe for what functions this method converges.

4. (a) State Lagrange Multiplier Rule for optimization problem

$$\text{minimize } f(x)$$
$$\text{subject to } h_1(x) = 0, \ldots, h_m(x) = 0$$

(b) Use Lagrange Multiplier Rule to find the rectangular box of the maximum volume inscribed into the ellipsoid

$$\{(x, y, z) \ : \ \frac{x^2}{a^2} + \frac{x^2}{b^2} + \frac{z^2}{c^2} = 1\}$$

Consider only boxes with sides which are parallel to the axes of the ellipsoid)

(c) Use second-order sufficiency conditions to verify optimality of your solutions

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5. State the definition of convex function. Let $f(x, u)$ be a convex function of both variables $(x, u) \in \mathbb{R}^n \times \mathbb{R}^m$ and the function

$$F(x) := \inf_{u \in \mathbb{R}^m} f(x, u)$$

is defined for all x. Show that $F(x)$ is convex function