MATH 6050

SAMPLE EXAM

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1. (a) State necessary first-order and second-order conditions for local minima and maxima of a function $f : \mathbb{R}^n \to \mathbb{R}$.

(b) Consider the problem

$$\text{minimize } f(x) = (x_1 - 2x_2)^2 + x_1^4$$

Find the minimizer of f. Determine that necessary second-order condition for a local minimizer is satisfied for this point? Is the second-order sufficient condition satisfied? Is this point a strict local minimizer? Is this point a global minimizer?

2. (a) Describe the Newton's method for unconstrained minimization of the function $f : \mathbb{R}^n \to \mathbb{R}$.

(b) For problem

$$\text{minimize } f(x) = x^4 - 1$$

perform one iteration of the newton's method starting from $x_0 = 4$.

(c) Consider problem minimize $f(x) = \frac{1}{2}x'Qx - c'x$, where Q is a positive-definite matrix. Prove that the Newton's method determines the minimizer of f in one iteration, regardlessly of the starting point.

3. (a) Give a definition of a convex set.

(b) Let C_1 and C_2 be convex sets. Is their intersection $C_1 \cap C_2$ convex set? Is their union $C_1 \cup C_2$ convex set? Justify your answers.

(c) Show that the following set

$$S := \{\mathbf{x} \in \mathbb{R}^n : \ A\mathbf{x} \geq \mathbf{b}\}$$

is a convex set (A is $m \times n$ matrix, $\mathbf{b} \in \mathbb{R}^m$).

4. (a) State a necessary first-order optimality conditions for the minimization problem with constraints and Mangasarian-Fromovitz Constraint Qualification Condition

$$\text{minimize } f(x)$$

$$\text{subject to} \qquad \boldsymbol{h}(x) = 0, \quad \boldsymbol{g}(x) \leq 0$$

where $x \in \mathbb{R}^n$ and $\boldsymbol{h} : \mathbb{R}^n \to \mathbb{R}^m$, $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^r$ are vector-functions.

(b) Consider the problem

$$\text{minimize } c'x$$

$$\text{subject to} \qquad Ax \leq b$$

where $c, x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ and A is a $m \times n$ matrix.

Write a first-order necessary conditions for a local minimum and show that any point x^* satisfying these conditions is a global minimizer.

5. (a) State Weak Duality Theorem for optimization problem

$$\text{minimize } f(x)$$
$$\text{subject to} \quad \boldsymbol{g}(x) \leq 0$$

(b) Find the dual problem to the following primal optimization problem

$$\text{minimize } f(x) = \frac{1}{2}x'Qx + c'x$$
$$\text{subject to} \quad Ax \leq b$$

where Q is a positive definite matrix.