5.1 Shell-and-Tube Heat Exchangers

The most common type of heat exchanger in industrial applications is shell-and-tube heat exchangers. The exchangers exhibit more than 65% of the market share with a variety of design experiences of about 100 years. Shell-and tube heat exchangers provide typically the surface area density ranging from 50 to 500 $\text{m}^2/\text{m}^3$ and are easily cleaned. The design codes and standards are available in the TEMA (1999)-Tubular Exchanger Manufacturers Association. A simple exchanger, which involves one shell and one pass, is shown in Figure 5.18.

Figure 5.18 Schematic of one-shell one-pass (1-1) shell-and-tube heat exchanger.

Baffles

In Figure 5.18, baffles are placed within the shell of the heat exchanger firstly to support the tubes, preventing tube vibration and sagging, and secondly to direct the flow to have a higher heat transfer coefficient. The distance between two baffles is *baffle spacing*.

Multiple Passes

Shell-and-tube heat exchangers can have multiple passes, such as 1-1, 1-2, 1-4, 1-6, and 1-8 exchangers, where the first number denotes the number of the shells and the second number denotes the number of passes. An odd number of tube passes is seldom used except the 1-1 exchanger. A 1-2 shell-and-tube heat exchanger is illustrated in Figure 5.19.
Figure 5.19 Schematic of one-shell two-pass (1-2) shell-and-tube heat exchanger.

![Schematic of one-shell two-pass (1-2) shell-and-tube heat exchanger](image)

Figure 5.20 Dimensions of 1-1 shell-and-tube heat exchanger

**Dimensions of Shell-and-Tube Heat Exchanger**

Some of the following dimensions are pictured in Figure 5.20.

- $L$ = tube length
- $N_t$ = number of tube
- $N_p$ = number of pass
- $D_s$ = Shell inside diameter
- $N_b$ = number of baffle
- $B$ = baffle spacing

The baffle spacing is obtained

$$B = \frac{L_t}{N_b + 1}$$  \hspace{1cm} (5.128)
Shell-Side Tube Layout

Figure 5.21 shows a cross section of both a square and triangular pitch layouts. The tube pitch $P_t$ and the clearance $C_t$ between adjacent tubes are both defined. Equation (5.30) of the equivalent diameter is rewritten here for convenience

$$D_e = \frac{4A_e}{P_{heated}} \quad (5.129)$$

From Figure 5.21, the *equivalent diameter* for the square pitch layout is

$$D_e = \frac{4\left(P_t^2 - \pi d_o^2 / 4\right)}{\pi d_o} \quad (5.130a)$$

From Figure 5.21, the *equivalent diameter* for the triangular pitch layout is

$$D_e = \frac{4\left(\sqrt{3}P_t^2 - \pi d_o^2 / 8\right)}{\pi d_o / 2} \quad (5.130b)$$

The cross flow area of the shell $A_c$ is defined as

$$A_c = \frac{D_s C_t B}{P_t} \quad (5.131)$$

![Flow Diagram](image)

Figure 5.21 (a) Square-pitch layout, (b) triangular-pitch layout.

The diameter ratio $d_r$ is defined by

$$d_r = \frac{d_o}{d_i} \quad (5.132)$$

Some diameter ratios for nominal pipe sizes are illustrated in Table C.6 in Appendix C. The tube pitch ratio $P_t$ is defined by
\[ P_t = \frac{P}{d_o} \] (5.133)

The tube clearance \( C_t \) is obtained from Figure 5.21.

\[ C_t = P_t - d_o \] (5.134)

The number of tube \( N_t \) can be predicted in fair approximation with the shell inside diameter \( D_s \).

\[ N_t = (CTP) \frac{\pi D_s^2 / 4}{\text{ShadeArea}} \] (5.135)

where \( CTP \) is the tube count constant that accounts for the incomplete coverage of the shell diameter by the tubes, due to necessary clearance between the shell and the outer tube circle and tube omissions due to tube pass lanes for multiple pass design [1].

\[
\begin{align*}
CTP &= 0.93 \quad \text{for one-pass exchanger} \\
CTP &= 0.9 \quad \text{for two-pass exchanger} \\
CTP &= 0.85 \quad \text{for three-pass exchanger}
\end{align*}
\] (5.136)

\[ \text{ShadeArea} = CL \cdot P_t^2 \] (5.137)

where \( CL \) is the tube layout constant.

\[
\begin{align*}
CL &= 1 \quad \text{for square-pitch layout} \\
CL &= \sin(60^\circ) = 0.866 \quad \text{for triangular-pitch layout}
\end{align*}
\] (5.138)

Plugging Equation (5.137) into (5.135) gives

\[ N_t = \frac{\pi}{4} \left( \frac{CTP}{CL} \right) \frac{D_s^2}{P_t^2} = \frac{\pi}{4} \left( \frac{CTP}{CL} \right) \frac{D_s^2}{P_t^2 d_o^2} \] (5.139)

Table 5.1 Summary of shell-and-tube heat exchangers

<table>
<thead>
<tr>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Equations</td>
<td>[ q = \dot{m}<em>1 c</em>{p1} (T_{i1} - T_{o1}) ] (5.140) [ q = \dot{m}<em>2 c</em>{p2} (T_{i2} - T_{o2}) ] (5.141)</td>
</tr>
<tr>
<td>Heat transfer areas of the inner and outer surfaces of an inner pipe</td>
<td>[ A_i = \pi \cdot d_i \cdot N_t \cdot L ] (5.142a) [ A_o = \pi \cdot d_o \cdot N_t \cdot L ] (5.142b)</td>
</tr>
</tbody>
</table>
Overall Heat Transfer Coefficient

\[
U_o = \frac{1/A_o}{ln\left(\frac{d_o}{d_i}\right)} + \frac{1}{h_i A_i + \frac{2\pi k L}{h_o A_o}}
\]  
(5.143)

**Tube side**

Reynolds number

\[
Re_D = \frac{\rho u_m d_i}{\mu} = \frac{\dot{m} d_i}{A_c \mu}
\]  
(5.144)

\[
A_c = \frac{\pi d_i^2 N_t}{4 N_p}
\]  
(5.144a)

Laminar flow (Re < 2300)

\[
Nu_D = \frac{hd_i}{k_f} = 1.86 \left(\frac{d_i \text{ Re Pr}}{L}\right)^{\frac{1}{4}} \left(\frac{\mu}{\mu_s}\right)^{0.14}
\]  
(5.145)

0.48 < Pr < 16,700

0.0044 < (\mu/\mu_s) < 9.75

Use \( Nu_D = 3.66 \) if \( Nu_D < 3.66 \)

Turbulent flow (Re > 2300)

\[
Nu_D = \frac{hd_i}{k_f} = \frac{(f/2)(\text{Re}_D - 1000)\text{Pr}}{1 + 12.7(f/2)^{1/2}(\text{Pr}^{2/3} - 1)}
\]  
(5.146)

3000 < Re_D < 5 \times 10^6 [4]

0.5 ≤ Pr ≤ 2000

Friction factor

\[
f = (1.58 \ln(\text{Re}_D) - 3.28)^2
\]  
(5.147)

**Shell side**

Square pitch layout (Figure 5.21)

\[
D_e = \frac{4\left(P_t^2 - \pi d_o^2/4\right)}{\pi d_o}
\]  
(5.148a)

Triangular pitch layout (Figure 5.21)

\[
D_e = \frac{4\left(\sqrt{3}P_t^2 - \pi d_o^2/8\right)}{\pi d_o}\n\]  
(5.148b)

Cross flow area

\[
A_e = \frac{D_e C_e B}{P_t}
\]  
(5.149)

Reynolds number

\[
\text{Re}_D = \frac{\rho u_m D_e}{\mu} = \frac{\dot{m} D_e}{A_c \mu}
\]  
(5.150)
Nusselt number

\[ Nu = \frac{h_d D_e}{k_f} = 0.36 \text{Re}^{0.55} \text{Pr}^{0.14} \]  

(5.151)

2000 < \text{Re} < 1 \times 10^6

**\( \varepsilon \)-NTU Method**

Heat transfer unit (NTU)

\[ NTU = \frac{U_o A_o}{(\dot{m}c_p)_{\text{min}}} \]  

(5.152)

Capacity ratio

\[ C_r = \frac{(\dot{m}c_p)_{\text{min}}}{(\dot{m}c_p)_{\text{max}}} \]  

(5.153)

Effectiveness

One shell (2, 4, ... passes)

\[ \varepsilon = 2 \left[ 1 + C_r + \left(1 + C_r^2\right)^{1/2} \right] \frac{1 + \exp\left[-NTU_1 \left(1 + C_r^2\right)^{1/2}\right]}{1 - \exp\left[-NTU_1 \left(1 + C_r^2\right)^{1/2}\right]} \]  

(5.154)

\[ NTU_1 = NTU / N_p \]

Heat transfer unit (NTU)

\[ NTU = -\left(1 + C_r^2\right)^{-1/2} \ln \left(\frac{E - 1}{E + 1}\right) \]  

(5.155)

where \( E = \frac{2}{\varepsilon} - \frac{1 + C_r}{(1 + C_r^2)^{1/2}} \)

Effectiveness

\[ \varepsilon = \frac{q}{q_{\text{max}}} = \frac{(\dot{m}c_p)_{\text{pl}} (T_{i_1} - T_{0_1})}{(\dot{m}c_p)_{\text{min}} (T_{i_1} - T_{21})} = \frac{(\dot{m}c_p)_{\text{pl}} (T_{2o} - T_{21})}{(\dot{m}c_p)_{\text{min}} (T_{i_1} - T_{21})} \]  

(5.156)

Heat transfer rate

\[ q = \varepsilon (\dot{m}c_p)_{\text{min}} (T_{i_1} - T_{21}) \]  

(5.157)

**Tube Side Pressure Drop**

Pressure drop

\[ \Delta P = 4 \left( \frac{f \cdot L_i}{d_i} + 1 \right) N_p \frac{1}{2} \rho \cdot v^2 \]  

(5.158)

Laminar flow

\[ f = 16 / \text{Re}_{D} \]  

(5.159)

Turbulent flow

\[ f = (1.58 \ln(\text{Re}_{D}) - 3.28)^2 \]  

(5.160)

**Shell Side Pressure Drop**
\[ \Delta P = f \frac{D_z}{D_e} (N_b + 1) \frac{1}{2} \rho \cdot v^2 \] (5.161)

\[ f = \exp(0.576 - 0.19 \ln(\text{Re}_s)) \] (5.162)
Example 5.2 Miniature Shell-and-Tube Heat Exchanger

A miniature shell-and-tube heat exchanger is designed to cool engine oil in an engine with the engine coolant (50% ethylene glycol). The engine oil at a flow rate of 0.23 kg/s enters the exchanger at 120°C and leaves at 115°C. The 50% ethylene glycol at a rate of 0.47 kg/s enters at 90°C. The tube material is Cr alloy ($k_w = 42.7 \text{ W/mK}$). Fouling factors of $0.176 \times 10^{-3} \text{ m}^2\text{K/W}$ for engine oil and $0.353 \times 10^{-3} \text{ m}^2\text{K/W}$ for 50% ethylene glycol are specified. Route the engine oil through the tubes. The permissible maximum pressure drop on each side is 10 kPa. The volume of the exchanger is required to be minimized. Since the exchanger is custom designed, the tube size can be smaller than NPS 1/8 (DN 6 mm) that is the smallest size in Table C.6 in Appendix C, wherein the tube pitch ratio of 1.25 and the diameter ratio of 1.3 can be applied. Design the shell-and-tube heat exchanger.

![Figure E5.2.1 Shell and tube heat exchanger](image)

**MathCAD format solution:**

Design concept is to develop a MathCAD modeling for a miniature shell-and-tube heat exchanger and then seek the solution by iterating the calculations by varying the parameters to satisfy the design requirements. It is reminded that the design requirements are the engine oil outlet temperature less than 115°C and the pressure drop less than 10 kPa in each side of the fluids.

The properties of engine oil and ethylene glycol are obtained using the average temperatures from Table C.5 in Appendix C.

\[
T_{\text{oil}} := \frac{(120°C + 115°C)}{2} = 117.5°C \quad \quad \quad T_{\text{cool}} := \frac{(90°C + 100°C)}{2} = 95°C
\]  
(E5.2.1)
Engine oil (subscript 1)-tube side 50% Ethylene glycol (subscript 2)-shell side

\[
\rho_1 := \frac{828 \text{ kg}}{\text{m}^3} \quad \rho_2 := \frac{1020 \text{ kg}}{\text{m}^3}
\] (E5.2.2)

\[
c_{p1} := \frac{2307 \text{ J}}{\text{kg} \cdot \text{K}} \quad \quad c_{p2} := \frac{3650 \text{ J}}{\text{kg} \cdot \text{K}}
\]

\[
k_1 := 0.135 \frac{\text{W}}{\text{m} \cdot \text{K}} \quad \quad k_2 := 0.442 \frac{\text{W}}{\text{m} \cdot \text{K}}
\]

\[
\mu_1 := 1.027 \times 10^{-2} \frac{\text{N} \cdot \text{s}}{\text{m}^2} \quad \quad \mu_2 := 0.08 \times 10^{-2} \frac{\text{N} \cdot \text{s}}{\text{m}^2}
\]

\[
Pr_1 := 175 \quad \quad Pr_2 := 6.6
\]

The thermal conductivity for the tube material (Chromium alloy) is given

\[
k_w := 42.7 \frac{\text{W}}{\text{m} \cdot \text{K}}
\] (E5.2.3)

**Given information:**

The inlet temperatures are given as

\[
T_{1i} := 120^\circ \text{C} \quad T_{2i} := 90^\circ \text{C}
\] (E5.2.4)

The mass flow rates are given as

\[
\dot{m}_{1} := 0.23 \frac{\text{kg}}{\text{s}} \quad \quad \dot{m}_{2} := 0.47 \frac{\text{kg}}{\text{s}}
\] (E.5.2.5)

The fouling factors for engine oil and 50% ethylene glycol are given as

\[
R_{fi} := 0.176 \times 10^{-3} \frac{\text{m}^2 \cdot \text{K}}{\text{W}} \quad R_{fo} := 0.353 \times 10^{-3} \frac{\text{m}^2 \cdot \text{K}}{\text{W}}
\] (E5.2.6)

**Design requirement:**

The engine oil outlet temperature must be less than 115°C.

\[
T_{1o} \leq 115^\circ \text{C}
\] (E5.2.7)

The pressure drop on each side must be

\[
\Delta P \leq 10 \text{kPa}
\] (E5.2.8)
**Design parameters to be sought by iterations**

Initially, estimate the following boxed parameters and then iterate the calculations with different values toward the design requirements.

- **Shell inside diameter** $D_s = 2.0\text{in}$
  \[D_s = 50.8\text{mm}\]  \hspace{1cm} (E5.2.9)

- **Tube length** $L_t = 15\text{in}$
  \[L_t = 381\text{mm}\]  \hspace{1cm} (E5.2.10)

- **Tube outside diameter** $d_o = \frac{1}{8}\text{in}$
  \[d_o = 3.175\text{mm}\]  \hspace{1cm} (E5.2.11)

The diameter ratio ($d_r = d_o/d_i$) is given as suggested in the problem description.

\[d_r := 1.3\quad d_i := \frac{1}{d_r}d_o\quad d_i = 2.442\text{mm}\]  \hspace{1cm} (E5.2.12)

The tube pitch ratio ($P_r = P_t/d_o$) is given as suggested in the problem description.

\[P_r := 1.25\]  \hspace{1cm} (E5.2.13)

The tube pitch is then obtained from Equation (5.133).

\[P_t := P_r\cdot d_o\]  \hspace{1cm} (E5.2.14)

The baffle spacing is assumed and may be iterated, and the baffle number from Equation (5.128) is defined.

\[B := 8\text{in}\quad B = 25.4\text{mm}\]  \hspace{1cm} (E5.2.15)

\[N_b := \frac{L_t}{B} - 1\quad N_b = 14\]  \hspace{1cm} (E5.2.16)

The number of passes is defined by

\[N_p := 1\]  \hspace{1cm} (E5.2.17)

The tube clearance $C_t$ is obtained from Figure 5.21 as

\[C_t := P_t - d_o\quad C_t = 0.794\text{mm}\]  \hspace{1cm} (E5.2.18)
From Equation (5.136), the tube count calculation constants (CTP) up to three-passes are given

$$CTP := \begin{cases} 
0.93 & \text{if } N_p = 1 \\
0.9 & \text{if } N_p = 2 \\
0.85 & \text{otherwise}
\end{cases}$$

(E5.2.19)

From Equation (5.138), the tube layout constant (CL) for a triangular-pitch layout is given by

$$CL := 0.866$$

(E5.2.20)

The number of tubes $N_t$ is estimated using Equation (5.139) and rounded off in practice. Note that the number of tubes in the shell inside diameter defined earlier indicates the compactness of a miniature exchanger. A 253-tube exchanger in a 2.25-inch shell outside diameter is commercially available for a 2-inch shell diameter.

$$N_{\text{tube}} \left( D_s, d_o, P_r \right) := \frac{\pi}{4} \cdot \frac{CTP}{CL} \cdot \frac{D_s^2}{P_r^2 \cdot d_o^2}$$

$$N_{\text{tube}} \left( D_s, d_o, P_r \right) = 138.189$$

(E5.2.21)

$$N_t := \text{round} \left( N_{\text{tube}} \left( D_s, d_o, P_r \right) \right)$$

$$N_t = 138$$

(E5.2.22)

**Tube side (Engine oil)**

The crossflow area, velocity and Reynolds number are defined as

$$A_{c1} := \frac{\pi \cdot d_i^2}{4} \cdot \frac{N_t}{N_p}$$

$$A_{c1} = 6.465 \times 10^{-4} \text{ m}^2$$

(E5.2.23)

$$v_1 := \frac{m_{\text{dot}}_1}{\rho_1 \cdot A_{c1}}$$

$$v_1 = 0.43 \text{ m/s}$$

(E5.2.24)

$$Re_1 := \frac{\rho_1 \cdot v_1 \cdot d_i}{\mu_1}$$

$$Re_1 = 84.603$$

(E5.2.25)

The Reynolds number indicates very laminar flow. The velocity in the tubes appears acceptable when considering a reasonable range of $0.5 – 1.0 \text{ m/s}$ in Table 5.4 for the engine oil.
The friction factor is determined automatically whether it is either laminar or turbulent using the following program as

\[
f(Re_D) := \begin{cases} 
(1.58 \ln(Re_D) - 3.28)^2 & \text{if } Re_D > 2300 \\
\frac{16}{Re_D} & \text{otherwise}
\end{cases}
\]

\[ (E5.2.26) \]

The Nusselt number for turbulent or laminar flow is defined using Equations (5.145) and (5.146) with assuming that \( \mu \) changes moderately with temperature. The convection heat transfer coefficient is then obtained.

\[
Nu_D(D_h, L_t, Re_D, Pr) := \begin{cases} 
\left( \frac{f(Re_D)}{2} \right)^2 \cdot \frac{(Re_D - 1000) \cdot Pr}{1 + 12.7 \left( \frac{f(Re_D)}{2} \right)^{0.5} \left( \frac{2}{Pr^3 - 1} \right)} \quad & \text{if } Re_D > 2300 \\
1.86 \left( \frac{D_h \cdot Re_D \cdot Pr}{L_t} \right)^{\frac{1}{3}} \quad & \text{otherwise}
\end{cases}
\]

\[ (E5.2.27) \]

\[
Nu_1 := Nu_D(d_i, L_t, Re_1, Pr_1) \\
Nu_1 = 8.484 \quad (E5.2.28)
\]

\[
h_1 := \frac{Nu_1 \cdot k_1}{d_i} \\
h_1 = 468.972 \frac{W}{m^2 \cdot K} \quad (E5.2.29)
\]

**Shell side (50% ethylene glycol)**

The free-flow area is obtained using Equation (5.131) and the velocity in the shell is also calculated.

\[
A_{c2} := \frac{D_s \cdot C_t \cdot B}{P_t} \\
A_{c2} = 2.581 \times 10^{-4} \, m^2 \quad (E5.2.30)
\]

\[
v_2 := \frac{m_{dot2}}{\rho_2 \cdot A_{c2}} \\
v_2 = 1.786 \frac{m}{s} \quad (E5.2.31)
\]
The velocity of 1.786 m/s in the shell is acceptable because the reasonable range of 1.2 – 2.4 m/s for the similar fluid shows in Table 5.4. The equivalent diameter for a triangular pitch is given in Equation (5.148b) as

\[
D_e := 4 \left[ \frac{P_t^2 \sqrt{3} - \frac{\pi \cdot d_o^2}{8}}{4 - \left( \frac{\pi \cdot d_o}{2} \right)^2} \right]
\]

\[
D_e = 2.295 \text{ mm}
\]

(E5.2.32)

\[
Re_2 := \frac{\rho_2 \cdot v_2 \cdot D_e}{\mu_2}
\]

\[
Re_2 = 5.225 \times 10^3
\]

(E5.2.33)

The Nusselt number is given in Equation (5.152) and the heat transfer coefficient is obtained.

\[
Nu_2 := 0.36 \cdot Re_2^{0.55} \cdot Pr_2^{0.3}
\]

(E5.2.34)

\[
h_2 := \frac{Nu_2 \cdot k_2}{D_e}
\]

\[
h_2 = 1.442 \times 10^4 \frac{W}{m^2 \cdot K}
\]

(E5.2.35)

The total heat transfer areas for both fluids are obtained as

\[
A_i := \pi \cdot d_i \cdot L_t \cdot N_t
\]

\[
A_i = 0.403 \text{ m}^2
\]

(E5.2.36)

\[
A_o := \pi \cdot d_o \cdot L_t \cdot N_t
\]

\[
A_o = 0.524 \text{ m}^2
\]

(E5.2.37)

The overall heat transfer coefficient is calculated using Equation (5.143) with the fouling factors as

\[
U_o := \frac{1}{A_o} \left( \frac{1}{h_1 \cdot A_i} + \frac{R_{fi}}{A_i} + \frac{\ln \left( \frac{d_o}{d_i} \right)}{2 \pi \cdot k_w \cdot L_t} + \frac{R_{fo}}{A_o} + \frac{1}{h_2 \cdot A_o} \right)
\]

\[
U_o = 209.677 \frac{W}{m^2 \cdot K}
\]

(E5.2.38)
ε-NTU method
The heat capacities for both fluids are defined and then the minimum and maximum heat capacities are obtained using the MathCAD built-in functions as

\[ C_1 := \text{mdot}_1 \cdot c_{p1} \quad C_1 = 530.61 \text{ W/K} \]  

(E5.2.39)

\[ C_2 := \text{mdot}_2 \cdot c_{p2} \quad C_2 = 1.716 \times 10^3 \text{ W/K} \]  

(E5.2.40)

\[ C_{\text{min}} := \min(C_1, C_2) \quad C_{\text{max}} := \max(C_1, C_2) \]  

(E5.2.41)

The heat capacity ratio is defined as

\[ C_r := \frac{C_{\text{min}}}{C_{\text{max}}} \quad C_r = 0.309 \]  

(E5.2.42)

The number of transfer unit is defined as

\[ \text{NTU} := \frac{U_o \cdot A_o}{C_{\text{min}}} \quad \text{NTU} = 0.207 \]  

(E5.2.43)

The effectiveness for shell-and-tube heat exchanger is give using Equation (5.154) as

\[ \text{NTU}_1 := \frac{\text{NTU}}{N_p} \]  

(E5.2.44a)

\[ \varepsilon_{\text{hx}} := 2 \left[ 1 + C_r + \left( 1 + C_r^2 \right)^{0.5} \cdot \frac{1 + \exp\left\{ -\text{NTU}_1 \left( 1 + C_r^2 \right)^{0.5} \right\}}{1 - \exp\left\{ -\text{NTU}_1 \left( 1 + C_r^2 \right)^{0.5} \right\}} \right]^{-1} \quad \varepsilon_{\text{hx}} = 0.182 \]  

(E5.2.44b)

Using Equation (5.156), the effectiveness is expressed as

\[ \varepsilon_{\text{hx}} = \frac{q}{q_{\text{max}}} = \frac{C_1 (T_{1i} - T_{1o})}{C_{\text{min}} (T_{1i} - T_{2i})} = \frac{C_2 (T_{2o} - T_{2i})}{C_{\text{min}} (T_{1i} - T_{2i})} \]  

(E5.2.45)

The outlet temperatures are rewritten for comparison with the outlet temperatures.
\( T_{1i} = 120^\circ C \) \hspace{1cm} \( T_{2i} = 90^\circ C \)

\[
T_{1o} := T_{1i} - \varepsilon_{hx} C_{min} \left( T_{1i} - T_{2i} \right) \hspace{1cm} T_{1o} = 114.544^\circ C \tag{E5.2.46}
\]

\[
T_{2o} := T_{2i} + \varepsilon_{hx} C_{min} \left( T_{1i} - T_{2i} \right) \hspace{1cm} T_{2o} = 91.687^\circ C \tag{E5.2.47}
\]

The engine oil outlet temperature of 114.544°C is close enough to the requirement of 105°C. The heat transfer rate is obtained

\[
q := \varepsilon_{hx} C_{min} \left( T_{1i} - T_{2i} \right) \hspace{1cm} q = 2.895 \times 10^3 \text{ W} \tag{E5.2.48}
\]

The pressure drops for both fluids are obtained using Equations (5.158) and (5.161) as

\[
\Delta P_1 := 4 \left( \frac{f(Re_1) L_t}{d_1} + 1 \right) N_p \frac{1}{2} \rho_1 v_1^2 \hspace{1cm} \Delta P_1 = 9.325 \text{ kPa} \tag{E5.2.49}
\]

\[
\Delta P_2 := f(Re_2) \frac{D_s}{D_c} \left( N_b + 1 \right) \frac{1}{2} \rho_2 v_2^2 \hspace{1cm} \Delta P_2 = 5.141 \text{ kPa} \tag{E5.2.50}
\]

Both the pressure drops calculated are less than the requirement of 10 kPa. The iteration between Equations (E5.2.9) and (E5.2.46) is terminated. The surface density \( \beta \) for the engine oil side is obtained using the relationship of the heat transfer area over the volume of the exchanger.

\[
\beta_1 := \frac{A_o}{\left( \frac{\pi D_s^2}{4} \right) L_t} \hspace{1cm} \beta_1 = 679.134 \frac{m^2}{m^3} \tag{E5.2.51}
\]

Summary of the design of the miniature shell-and-tube heat exchanger

Given information

\( T_{1i} = 120^\circ C \) \hspace{1cm} \text{ engine oil inlet temperature } \)

\( T_{2i} = 90^\circ C \) \hspace{1cm} \text{ 50\% ethylene glycol inlet temperature }
\[
\text{mdot}_1 = 0.23 \frac{\text{kg}}{\text{s}} \quad \text{mass flow rate of engine oil}
\]
\[
\text{mdot}_2 = 0.47 \frac{\text{kg}}{\text{s}} \quad \text{mass flow rate of 50\% ethylene glycol}
\]
\[
R_{fi} = 1.76 \times 10^{-4} \cdot \frac{\text{m}^2}{\text{K W}} \quad \text{fouling factor of engine oil}
\]
\[
R_{fo} = 3.53 \times 10^{-4} \cdot \frac{\text{m}^2}{\text{K W}} \quad \text{fouling factor of 50\% ethylene glycol}
\]

Requirements for the exchanger

\[ T_{1o} \leq 115^\circ \text{C} \quad \text{Engine outlet temperature} \]
\[ \Delta P_1 \leq 10\text{kPa} \quad \text{Pressure drop on both sides} \]

Design obtained

\[
N_p = 1 \quad \text{number of passes}
\]
\[
D_s = 50.8 \text{ mm} \quad \text{shell inside diameter} \quad D_s = 2 \text{ in}
\]
\[
d_o = 3.175 \text{ mm} \quad \text{tube outer diameter}
\]
\[
d_i = 2.442 \text{ mm} \quad \text{tube inner diameter}
\]
\[
L_t = 381 \text{ mm} \quad \text{tube length} \quad L_t = 15 \text{ in}
\]
\[
N_t = 138 \quad \text{number of tube}
\]
\[
C_t = 0.794 \text{ mm} \quad \text{tube clearance}
\]
\[
B = 25.4 \text{ mm} \quad \text{baffle spacing} \quad B = 1 \text{ in}
\]
\[
N_b = 14 \quad \text{number of baffle}
\]
\[
T_{1o} = 114.544^\circ \text{C} \quad \text{engine oil outlet temperature}
\]
\[
T_{2o} = 91.687^\circ \text{C} \quad \text{50\% ethylene glycol outlet temperature}
\]
\[
q = 2.895 \text{ kW} \quad \text{heat transfer rate}
\]
\[
\beta_1 = 679 \cdot \frac{\text{m}^2}{\text{m}^3} \quad \text{surface density}
\]
\[
\Delta P_1 = 9.325 \text{ kPa} \quad \text{pressure drop for engine oil}
\]
\[ \Delta P_2 = 5.141 \, \text{kPa} \] pressure drop for 50\% ethylene glycol

The design satisfies the requirements.

Problems (corrected)

Shell-and-Tube Heat Exchanger

5.3 A miniature shell-and-tube heat exchanger is designed to cool glycerin with cold water. The glycerin at a flow rate of 0.25 kg/s enters the exchanger at 60°C and leaves at 50°C. The water at a rate of 0.54 kg/s enters at 18°C, which is shown in Figure P5.3. The tube material is Cr alloy \((k_w = 60.5 \, \text{W/mK})\). Fouling factors of \(0.253 \times 10^{-3} \, \text{m}^2\text{K/W}\) for water and \(0.335 \times 10^{-3} \, \text{m}^2\text{K/W}\) for glycerin are specified. Route the glycerin through the tubes. The permissible maximum pressure drop on each side is 30 kPa. The volume of the exchanger is required to be minimized. Since the exchanger is custom designed, the tube size may be smaller than NPS 1/8 (DN 6 mm) that is the smallest size in Table C.6 in Appendix C, wherein the tube pitch ratio of 1.25 and the diameter ratio of 1.3 can be applied. Design the shell-and-tube heat exchanger.

Figure P5.3 Shell-and tube heat exchanger