Thermoelectric Coolers

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1. Introduction

Thermoelectric coolers have comprehensive applications [1-5] in electronic devices, medical instruments, automotive air conditioners, and refrigerators. Since the discovery of thermoelectric effects in the early nineteenth century, a very essential equation for the rate of heat flow per unit area is given [18-20]

$$\vec{q} = \alpha T\vec{j} - k\vec{\nabla}T \tag{1}$$

This equation relates the electric current and the thermal conduction, and finally leads to the steady-state heat diffusion equation:

$$\vec{\nabla}\cdot\left(k\vec{\nabla}T\right) + j^2\rho - T\frac{d\alpha}{dT}\vec{j}\cdot\vec{\nabla}T = 0 \tag{2}$$

where the first term gives the thermal conduction, the second term gives the Joule heating, and the third term pertains to the Thomson effect which results from the temperature-dependent Seebeck coefficient. The above two equation governs the thermoelectric phenomena.

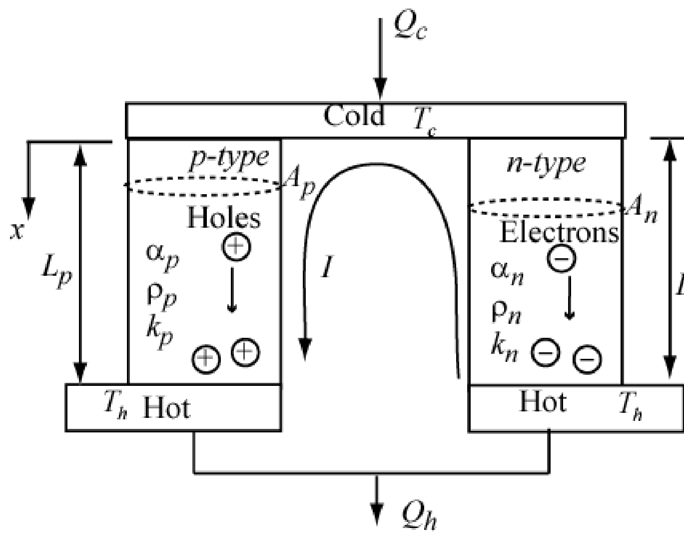


Figure 1. Thermoelectric cooler with p-type and n-type thermoelements.

Consider a one-dimensional p-type and n-type thermocouple with length L and cross-sectional area A as shown in Figure 1. With an assumption that the Seebeck coefficient is independent of temperature, Equation (2) reduces to

$$\frac{d}{dx}\left(kA\frac{dT}{dx}\right)+\frac{I^2\rho}{A}=0 \tag{3}$$

The solution for the temperature gradient with two boundary conditions ($T_{x=0}=T_c$ and $T_{x=L}=T_h$) is

$$\left.\frac{dT}{dx}\right|_{x=0}=\frac{I^2\rho L}{2A^2k}+\frac{T_h-T_c}{L} \tag{4}$$

Equation (1) is expressed in terms of p-type and n-type thermoelements.

$$\dot{Q}_c=(\alpha_p-\alpha_n)T_cI+\left(-kA\left.\frac{dT}{dx}\right|_{x=0}\right)_p+\left(-kA\left.\frac{dT}{dx}\right|_{x=0}\right)_n \tag{5}$$

where $\dot{Q}_c$ is the rate of heat absorbed at the cold junction. Substituting Equation (4) in (5) gives

$$\dot{Q}_c=(\alpha_p-\alpha_n)T_cI-\frac{1}{2}I^2\left(\frac{\rho_pL_p}{A_p}+\frac{\rho_nL_n}{A_n}\right)-\left(\frac{k_pA_p}{L_p}+\frac{k_nA_n}{L_n}\right)(T_h-T_c) \tag{6}$$

Finally, the cooling power with n thermocouples at the junction of temperature T_c is expressed as

$$\dot{Q}_c=n\left(\alpha T_cI-\frac{1}{2}I^2R-K\Delta T\right) \tag{7}$$

where

$$\alpha=\alpha_p-\alpha_n \tag{8}$$

$$R=\frac{\rho_pL_p}{A_p}+\frac{\rho_nL_n}{A_n} \tag{9}$$

$$K=\frac{k_pA_p}{L_p}+\frac{k_nA_n}{L_n} \tag{10}$$

$$\Delta T = T_h - T_c \tag{11}$$

If we assume that p-type and n-type thermocouples are similar, we have that $R = \rho L/A$ and $K = kA/L$, where $\rho = \rho_p + \rho_n$ and $k = k_p + k_n$. Equation (7) is called the ideal equation which has been widely used in science and engineering. The rate of heat liberated at the hot junction with n thermocouples is

$$\dot{Q}_h = n\left(\alpha T_h I + \frac{1}{2} I^2 R - K\Delta T\right) \tag{12}$$

Considering the 1st law of thermodynamics across the thermoelectric device, we have

$$\dot{W}_n = \dot{Q}_h - \dot{Q}_c \tag{13}$$

The amount of work per unit time across the thermoelement couple is obtained using Equations (7) and (12) in (13).

$$\dot{W}_n = n\left[\alpha I\left(T_h - T_c\right) + I^2 R\right] \tag{14}$$

where the first term is the rate of work to overcome the thermoelectric voltage, whereas the second term is the resistive loss. Since the power is $\dot{W}_n = IV_n$, the voltage across the couple will be

$$V_n = n\left[\alpha\left(T_h - T_c\right) + IR\right] \tag{15}$$

The coefficient of performance (COP) is defined by the ratio of the cooling power to the input electrical power.

$$COP = \frac{\dot{Q}_c}{\dot{W}_n} = \frac{n\left(\alpha T_c I - \frac{1}{2} I^2 R - K\Delta T\right)}{n\left(\alpha I\Delta T + I^2 R\right)} = \frac{\alpha T_c I - \frac{1}{2} I^2 R - K\Delta}{\alpha I\Delta T + I^2 R} \tag{16}$$

There are two values of the current that are of special interest: the current I_{mp} that yields the maximum cooling power and the current I_{COP} that yields the maximum COP. The maximum cooling power can be obtained by differentiating Equation (7) and setting it to zero. The current for the maximum cooling power is found to be

$$I_{mp} = \frac{\alpha T_c}{R} \tag{17}$$

The maximum COP can be obtained by differentiating Equation (16) and setting it to zero. We finally have

$$I_{COP} = \frac{\alpha \Delta T}{R\left(\sqrt{1+Z\overline{T}}-1\right)} \tag{18}$$

where $Z = \alpha^2 / \rho k$ or, equivalently, $Z = \alpha^2 / RK$ which is called the figure of merit and $\overline{T}$ is the average temperature and $Z\overline{T}$ is expressed by $Z\overline{T} = Z\left(T_h + T_c\right)/2$. The maximum COP is expressed by

$$COP_{\max} = \frac{T_c}{T_h - T_c} \frac{\left(1+Z\overline{T}\right)^{\frac{1}{2}} - \frac{T_h}{T_c}}{\left(1+Z\overline{T}\right)^{\frac{1}{2}} + 1} \tag{19}$$

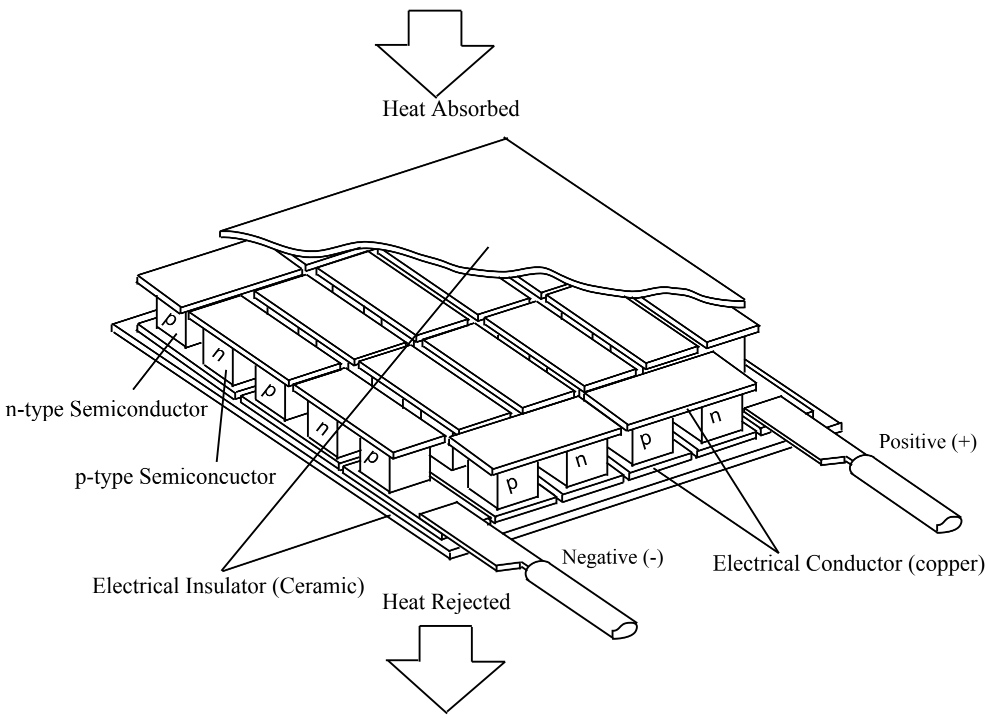


Figure 2. Cutaway of a typical thermoelectric module

1. Theoretical Maximum Parameters

Let us consider a thermoelectric module shown in Figure 2 for the theoretical maximum parameters with the ideal equation. The module consists of a number of thermoelement couples as shown. As mentioned before, the ideal equation assumes that there are no electrical and thermal contact resistances, no Thomson effect, and no radiation or convection. We hereunder derive the theoretical maximum parameters useful.

The maximum current I_{max} is the current that produces the maximum possible temperature difference ΔT_{max}, which always occurs when the cooling power is at zero.

This is obtained by setting $\dot{Q}_c$ = 0 in Equation (7), replacing T_c with (T_h – ΔT) and taking derivative of ΔT with respect to I and setting it to zero. The maximum current is finally expressed by

$$I_{\max} = \frac{\alpha}{R}\left(\sqrt{\left(T_h + \frac{1}{Z}\right)^2 - T_h^{\ 2}} - \frac{1}{Z}\right) \tag{20}$$

Or, equivalently in terms of ΔT_{max},

$$I_{\max} = \frac{\alpha\left(T_h - \Delta T_{\max}\right)}{R} \tag{21}$$

The maximum temperature difference ΔT_{max} is the maximum possible temperature difference ΔT_{max} , which always occurs when the cooling power is at zero and the current is at maximum. This is obtained by setting $\dot{Q}_c$ = 0 in Equation (7), substituting both I and T_c by I_{max} and T_h – ΔT_{max}, respectively, and solving for ΔT_{max}. The maximum temperature difference is obtained as

$$\Delta T_{\max} = \left(T_h + \frac{1}{Z}\right) - \sqrt{\left(T_h + \frac{1}{Z}\right)^2 - T_h^{\ 2}} \tag{22}$$

where the figure of merit Z (unit: K^{-1}) is given by

$$Z = \frac{\alpha^2}{\rho k} \quad \text{also} \quad Z = \frac{\alpha^2}{RK} \tag{23}$$

The maximum cooling power $\dot{Q}_{c\max}$ is the maximum thermal load which occurs simultaneously at $\Delta T = 0$ and $I = I_{max}$. This can be obtained by substituting both I and T_c in Eqiuation (7) by I_{max} and T_h, respectively, and solving for $\dot{Q}_{c\max}$. The maximum cooling power for a thermoelectric module with n thermoelement couples is

$$\dot{Q}_{c\max} = \frac{n\alpha^2\left(T_h^{\ 2} - \Delta T_{\max}^{\ \ 2}\right)}{2R} \tag{24}$$

The maximum voltage is the DC voltage which delivers the maximum possible temperature difference ΔT_{max} when $I = I_{max}$. The maximum voltage is obtained from Equations (15) and (21), which is

$$V_{\max} = n\alpha T_h \tag{25}$$

Note that there are four inherent maximum parameters, which are the maximum current, maximum temperature difference, maximum cooling power, and maximum voltage. Also note that the four maximum parameters are expressed as a function of three material properties (α, ρ, and k) with the given geometry of thermoelements (A/L and n).

2. Normalized Parameters

If we divide the active values by the maximum values, we can normalize the characteristics of the thermoelectric cooler. The normalized cooling power can be obtained by dividing Equation (7) by Equation (24), which is

$$\frac{\dot{Q}_c}{\dot{Q}_{c\max}} = \frac{n\left(\alpha\left(T_h - \Delta T\right)I - \frac{1}{2}I^2 R - K\Delta T\right)}{n\alpha^2\left(T_h^{\ 2} - \Delta T_{\max}^{\ \ 2}\right)/2R} \tag{26}$$

which, in terms of the normalized current and normalized temperature difference, reduces to

$$\frac{\dot{Q}_c}{\dot{Q}_{c\max}} = \frac{2\left(1 - \frac{\Delta T}{\Delta T_{\max}}\frac{\Delta T_{\max}}{T_h}\right)\frac{I}{I_{\max}}}{1 + \frac{\Delta T_{\max}}{T_h}} - \frac{\left(1 - \frac{\Delta T_{\max}}{T_h}\right)\left(\frac{I}{I_{\max}}\right)^2}{1 + \frac{\Delta T_{\max}}{T_h}} - \frac{2\frac{\Delta T}{\Delta T_{\max}}\frac{\Delta T_{\max}}{T_h}}{ZT_h\left[1 - \left(\frac{\Delta T_{\max}}{T_h}\right)^2\right]} \tag{27}$$

where

$$\frac{\Delta T_{\max}}{T_h} = \left(1 + \frac{1}{ZT_h}\right) - \sqrt{\left(1 + \frac{1}{ZT_h}\right)^2 - 1} \tag{28}$$

The coefficient of performance in terms of the normalized values is

$$COP = \frac{\left(1 - \frac{\Delta T}{\Delta T_{\max}}\frac{\Delta T_{\max}}{T_h}\right)\frac{I}{I_{\max}} - \frac{1}{2}\left(1 - \frac{\Delta T_{\max}}{T_h}\right)\left(\frac{I}{I_{\max}}\right)^2 - \frac{\frac{\Delta T}{\Delta T_{\max}}\frac{\Delta T_{\max}}{T_h}}{ZT_h\left(1 - \frac{\Delta T_{\max}}{T_h}\right)}}{\frac{\Delta T}{\Delta T_{\max}}\frac{\Delta T_{\max}}{T_h}\frac{I}{I_{\max}} + \left(1 - \frac{\Delta T_{\max}}{T_h}\right)\left(\frac{I}{I_{\max}}\right)^2} \tag{29}$$

The normalized voltage is

$$\frac{V}{V_{\max}} = \frac{\Delta T}{\Delta T_{\max}}\frac{\Delta T_{\max}}{T_h} + \left(1 - \frac{\Delta T_{\max}}{T_h}\right)\frac{I}{I_{\max}} \tag{30}$$

The normalized current for the optimum *COP* is obtained from Equation (18).

$$\frac{I_{COP}}{I_{\max}} = \frac{\dfrac{\Delta T}{\Delta T_{\max}} \dfrac{\Delta T_{\max}}{T_h}}{\left(1 - \dfrac{\Delta T_{\max}}{T_h}\right)\left(\sqrt{1 + Z\overline{T}} - 1\right)} \tag{31}$$

where $Z\overline{T}$ is expressed on basis of T_h by

$$Z\overline{T} = ZT_h\left(1 - \frac{1}{2}\left(\frac{\Delta T_{\max}}{T_h}\right)\left(\frac{\Delta T}{\Delta T_{\max}}\right)\right) \tag{32}$$

Note that the above normalized values in Equations (27), (29) and (30) are functions only of three parameters, which are $\Delta T/\Delta T_{\max}$, $I/I_{\max}$ and ZT_h.

3. Effective Material Properties

As mentioned before, theoretically, the four maximum parameters (I_{max}, ΔT_{max}, $\dot{Q}_{c\max}$ and V_{max}) are exactly reciprocal with the three material properties (α, ρ, and k). In other words, the three material properties constitute the four maximum parameters in a reciprocal manner. In order to predict the performance of thermoelectric coolers, the material properties are, of course, required. However, we have a dilemma that usually manufacturers do not provide the material properties as their proprietary information but the measured maximum parameters as specifications of their products. Using the reciprocal relationship, we can easily formulate the three material properties in terms of the four manufacturers' maximum parameters. Two maximum parameters (I_{max} and ΔT_{max}) are essential and must be used, but there is a choice that either $\dot{Q}_{c\max}$ or V_{max} is selected. Theoretically there is no difference whether either is selected but practically there is a difference depending on the choice. According to the analysis (not shown here), if we choose the maximum cooling power, the errors between the ideal equation and real measurements tend to go to the voltages. On the other hand, if we choose the maximum voltage, the errors tend to be distributed evenly to the cooling powers and voltages. It should be noted that there is no longer the reciprocity between the four maximum parameters and the three material properties if we determine the material properties by extracting them from the manufacturers' maximum parameters. The material properties extracted are called the effective material properties.

The effective figure of merit is obtained from Equation (22), which is

$$Z^* = \frac{2\Delta T_{\max}}{\left(T_h - \Delta T_{\max}\right)^2} \tag{33}$$

The effective Seebeck coefficient is obtained using Equations (21) and (24), which is

$$\alpha^* = \frac{2\dot{Q}_{c\max}}{nI_{\max}\left(T_h + \Delta T_{\max}\right)} \quad (34)$$

The effective electrical resistivity can be obtained using Equation (21), which is

$$\rho^* = \frac{\alpha^*\left(T_h - \Delta T_{\max}\right)A/L}{I_{\max}} \quad (35)$$

The effective thermal conductivity is now obtained using Equation (23), which is

$$k^* = \frac{\alpha^{*2}}{\rho^* Z^*} \quad (36)$$

The effective material properties include effects such as the electrical and thermal contact resistances, the temperature dependency of the material, and the radiative and convective heat losses. Hence, the effective figure of merit appears slightly smaller than the intrinsic figure of merit as shown in Table 1. Since the material properties were obtained for a p-type and n-type thermoelement couple, the material properties of a thermoelement (either p-type or n-type) should be attained by dividing it by 2.

Table 1 Comparison of the Properties and Dimensions for the Commercial Products of Thermoelectric Modules

Description	TEC Module (Bismuth Telluride)				
	Symbols	Laird CP10-127-05 (T_h=298 K)	Marlow RC12-4 (T_h=298 K)	Kryotherm TB-127-1.0-1.3 (T_h=298 K)	Tellurex C2-30-1503 (T_h=300 K)
# of thermocouples	n	127	127	127	127
Intrinsic material properties (provided by manufacturers)	α (μV/K)	202.17	-	-	-
	ρ (Ωcm)	1.01×10^{-3}	-	-	-
	k (W/cmK)	1.51×10^{-2}	-	-	-
	ZT_h	0.803	-	-	-
Effective material properties (calculated using commercial ΔT_{max}, I_{max}, and Q_{cmax})	α* (μV/K)	189.2	211.1	204.5	208.5
	ρ* (Ωcm)	0.9×10^{-3}	1.15×10^{-3}	1.0×10^{-3}	1.0×10^{-3}
	k* (W/cmK)	1.6×10^{-2}	1.7×10^{-2}	1.6×10^{-2}	1.7×10^{-2}
	ZT_h*	0.744	0.673	0.776	0.758
Measured geometry of thermoelement	A (mm^2)	1.0	1.0	1.0	1.21
	L (mm)	1.25	1.17	1.3	1.66
	$G=A/L$ (cm)	0.080	0.085	0.077	0.073
Dimension (W×L×H)	mm	30 × 30 × 3.2	30 × 30 × 3.4	30 × 30 × 3.6	30 × 30 × 3.7
Manufacturers' maximum parameters	ΔT_{max} (°C)	67	66 (63)	69	68
	I_{max} (A)	3.9	3.7	3.6	3.5
	Q_{cmax} (W)	34.3	36	34.5	34.1
	V_{max} (V)	14.4	14.7	15.7	15.5
	R (Ω)-module	3.36	3.2	3.2	3.85
Effective maximum	ΔT_{max} (°C)	67	63	69	68

parameters (calculated using $\alpha*$, $\rho*$, and $k*$	I_{max} (A)	3.9	3.7	3.6	3.5
	Q_{cmax} (W)	34.3	36	34.5	34.1
	V_{max} (V)	14.37	16.08	15.58	15.88
	R (Ω)-module	2.86	3.43	3.33	3.51

System designers using thermoelectric coolers usually meet two practical problems. One is that the ideal equation inherently involves three errors which are the electrical and thermal contact resistances, the Thomson effect, and the radiation and convection heat transfer. Among them, the contact resistance is believed to be a primary source of errors according to the reports [6, 9, 10, 12]. The other is that the manufacturers usually do not provide the material properties for their products, leaving designers with some degree of difficulty as to which modules should be used. The present method determines the effective material properties of $\alpha*$, $\rho*$, and $k*$ using the three maximum parameters of ΔT_{max}, I_{max}, and $\dot{Q}_{c\max}$ out of the four parameters of ΔT_{max}, I_{max}, $\dot{Q}_{c\max}$, and V_{max}. In summary, the present method using the ideal equation with the effective material properties gives reasonable accuracy when comparing predictions from the ideal equation to the four major manufacturers' performance curve data.

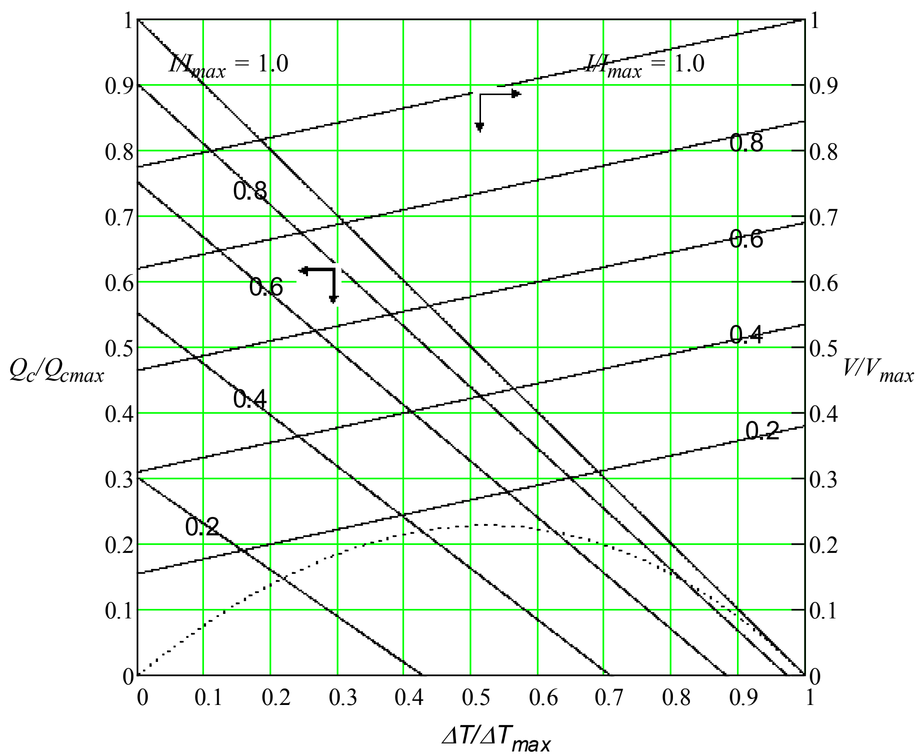


Figure 3. Normalized chart I: cooling power and voltage versus ΔT as a function of current. The solid lines depict the data at ZT_h = 0.75. The dashed line depicts the cooling power ratios at the optimum COP.

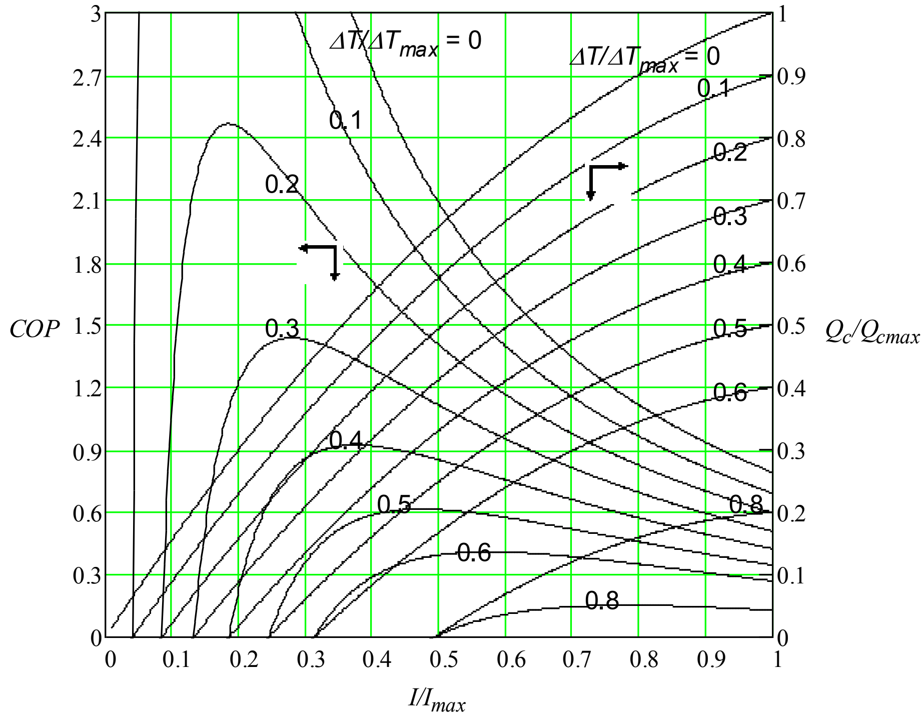


Figure 4. Normalized chart II: cooling power and *COP* versus current as a function of ΔT. The solid lines depict the data at $ZT_h = 0.75$.

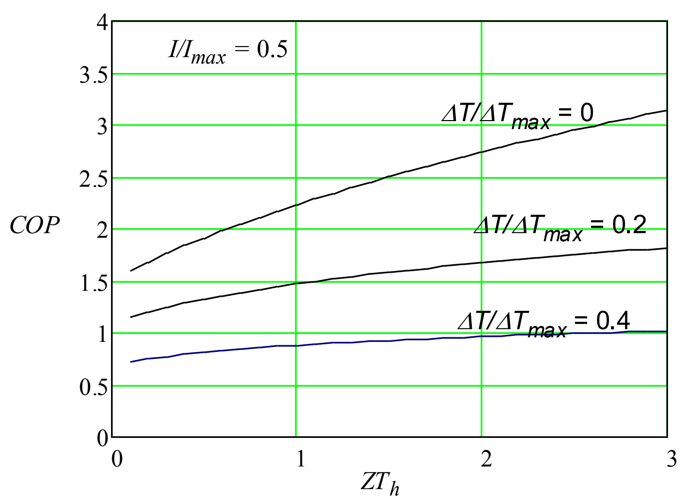


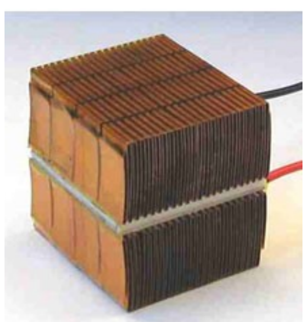
Figure 5. *COP* versus ZT_h as a function of $\Delta T/\Delta T_{\max}$ for $I/I_{\max} = 0.5$.

Equations (27), (29) and (30) were used to generate Figures 3 and 4, which are completely based on the ideal equation, presenting the thermal and electrical characteristics of thermoelectric coolers. Keeping in mind that the three maximum parameters of ΔT_{max}, I_{max}, and $\dot{Q}_{c\max}$ are exactly predictable with the effective material properties, we can accordingly examine the normalized charts I and II of Figures 3 and 4. The solid lines for the both figures indicate the normalized prediction at $ZT_h = 0.75$, which is approximately a typical commercial value as shown in Table 1. We can see three characteristics from the two figures. First of all, regardless of the effective material properties, the normalized cooling power $\dot{Q}_c/\dot{Q}_{c\max}$ is a relatively weak function of ZT_h while both the normalized voltage $V/V_{\max}$ and the *COP* are an appreciable function of ZT_h. Second, these normalized charts may be used to predict the realistic quantities by simply substituting the manufacturer's maximum parameters for the maximum parameters in the charts. In other words, the normalized charts would be universal for any commercial module if an appropriate ZT_h is used to compensate for the errors prevalent in the normalized voltage. It is seen in Figure 4 that the maximum *COPs* show the impractically low cooling powers, especially when $\Delta T/\Delta T_{\max}$ lies between 0 and 0.5 as in the typical air conditioners and refrigerators. Therefore, the preferred normalized current would be somewhere between maximum cooling power and maximum *COP* saying approximately at $I/I_{\max} = 0.5$. The effect of ZT_h on the *COP* is shown in Figure 5 for $I/I_{\max} = 0.5$. In order to compete with the conventional *COP* ≈ 0.25 when $\Delta T/\Delta T_{\max} \approx 0$, the material having ZT_h = 3 should be developed.

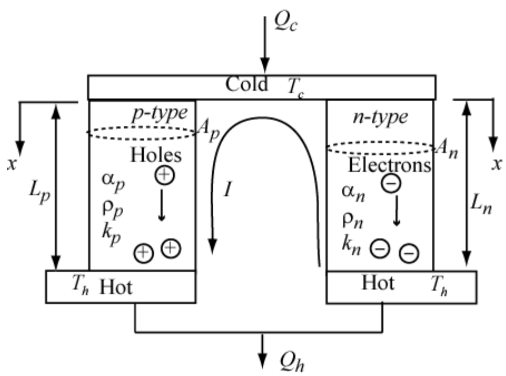
Example E-2

A novel thermoelectric air conditioner is designed as a part of green energy application for replacement of the conventional compressor-type air conditioner in a car. A thermoelectric module with heat sinks (Figure E-2a) consists of n = 128 p- and n-type thermocouples, one of which is shown in Figure E-2b. The air conditioner has a number of the modules. Cabin cold air enters the upper heat sink in Figure E-2a, while the outside ambient air enters the lower heat sink. An electric current is applied in a way that a heat flow (cooling power) should be absorbed at the cold junction temperature of 15 °C and liberated at the hot junction temperature of 40 °C (Figure E-2b). The TEC material of Bismuth telluride (Bi_2Te_3) is used having the properties as $\alpha_p = -\alpha_n = 200$ μV/K, $\rho_p = \rho_n = 1.0 \times 10^{-3}$ Ωcm, and $k_p = k_n = 1.52 \times 10^{-2}$ W/cmK. The cross-sectional area and leg length of the thermoelement are $A_n = A_p = 2$ mm^2 and $L_n = L_p = 1$ mm, respectively. Assuming that the cold and high junction temperatures are steadily maintained, answer the following questions (Use hand calculations).

(a) For the maximum cooling power, compute the current, cooling power, and *COP*.
(b) For the maximum *COP*, compute the current, cooling power, and *COP*.
(c) If the midpoint of the current between the maximum cooling power and maximum *COP* is used for the optimal design, compute the current, the cooling power and *COP*.
(d) If the total cooling load of 630 W (per occupant) for the air conditioner is required, compute the number of modules to meet the requirement using the midpoint of current.



(a)



(b)

Figure E-2. (a) A thermoelectric module. (b) A p-type and n-type thermocouple

Solution:

Material properties: $\alpha = \alpha_p - \alpha_n = 400 \times 10^{-6}$ V/K, $\rho = \rho_p + \rho_n = 2.0 \times 10^{-5}$ Ωm, and $k = k_p + k_n = 3.04$ W/mK

The number of thermocouples is n = 128. The hot and cold junction temperatures are

$$T_h = (40 + 273)K = 313K \text{ and } T_c = (15 + 273)K = 288K$$

$$\Delta T = T_h - T_c = 25K$$

The figure of merit is

$$Z = \frac{\alpha^2}{\rho k} = \frac{\left(400 \times 10^{-6}\, V/K\right)^2}{\left(2.0 \times 10^{-5}\,\Omega m\right)\left(3.04\, W/mK\right)} = 2.632 \times 10^{-3}\, K^{-1}$$

and the dimensionless figure of merit is

$$ZT_c = \left(2.632 \times 10^{-3}\, K^{-1}\right)\left(288K\right) = 0.758$$

The internal resistance R and the thermal conductance K are calculated as

$$R = \frac{\rho L}{A} = \frac{\left(2.0 \times 10^{-5}\,\Omega m\right)\left(1 \times 10^{-3}\, m\right)}{2 \times 10^{-6}\, m^2} = 0.01\Omega$$

$$K = \frac{kA}{L} = \frac{\left(3.04W/mK\right)\left(2 \times 10^{-6}\, m^2\right)}{1 \times 10^{-3}\, m} = 6.08 \times 10^{-3}\, \frac{W}{K}$$

(a) For the maximum cooling power:

Using Equation (17), the current for the maximum cooling power is

$$I_{mp} = \frac{\alpha T_c}{R} = \frac{\left(400 \times 10^{-6}\, V/K\right)\left(288K\right)}{\left(0.01\Omega\right)} = 11.526A$$

Using Equation (7), the maximum cooling power is

$$\begin{aligned} \dot{Q}_{cmp} &= n\left(\alpha T_c I_{mp} - \frac{1}{2} I_{mp}{}^2 R - K\Delta T\right) \\ &= \left(128\right)\left[\left(400 \times 10^{-6}\, V/K\right)\left(288K\right)\left(11.526A\right) - \frac{1}{2}\left(11.526A\right)^2\left(0.01\Omega\right) - \left(6.08 \times 10^{-3}\, \frac{W}{K}\right)\left(25K\right)\right] \\ &= 65.567W \end{aligned}$$

Using Equation (14), the power input is

$$\begin{aligned} \dot{W}_{nmp} &= n\left[\alpha I_{mp}\left(T_h - T_c\right) + I_{mp}{}^2 R\right] \\ &= \left(128\right)\left[\left(400 \times 10^{-6}\, V/K\right)\left(11.526A\right)\left(25K\right) + \left(11.526A\right)^2\left(0.01\Omega\right)\right] = 184.8W \end{aligned}$$

Using Equation (16), the COP at the maximum cooling power is

$$COP_{mp} = \frac{\dot{Q}_{cmp}}{\dot{W}_{nmp}} = \frac{65.567W}{184.8W} = 0.355$$

(b) For the maximum *COP*:

$$Z\overline{T} = Z\frac{T_h + T_c}{2} = \left(2.632\times10^{-3}K^{-1}\right)\frac{25K}{2} = 0.791$$

Using Equation (18), the current for the maximum *COP* is

$$I_{COP} = \frac{\alpha\Delta T}{R\left(\sqrt{1+Z\overline{T}}-1\right)} = \frac{\left(400\times10^{-6}\,V/K\right)\left(25K\right)}{\left(0.01\Omega\right)\left(\sqrt{1+0.791}-1\right)} = 2.956A$$

Using Equation (7), the maximum cooling power is

$$\dot{Q}_{ncop} = n\left(\alpha T_c I_{cop} - \frac{1}{2}I_{cop}{}^2 R - K\Delta T\right)$$

$$= \left(128\right)\left[\left(400\times10^{-6}\,V/K\right)\left(288K\right)\left(2.956A\right) - \frac{1}{2}\left(2.956A\right)^2\left(0.01\Omega\right) - \left(6.08\times10^{-3}\,\frac{W}{K}\right)\left(25K\right)\right]$$

$$= 18.557W$$

Using Equation (14), the maximum power input is

$$\dot{W}_{ncop} = n\left[\alpha I_{cop}\left(T_h - T_c\right) + I_{cop}{}^2 R\right]$$

$$= \left(128\right)\left[\left(400\times10^{-6}\,V/K\right)\left(2.956A\right)\left(25K\right) + \left(2.956A\right)^2\left(0.01\Omega\right)\right] = 14.964W$$

Using Equation (16), the *COP* is

$$COP_{\max} = \frac{\dot{Q}_{ncop}}{\dot{W}_{ncop}} = \frac{18.557W}{14.964W} = 1.24$$

(c) For the midpoint of the current between the maximum cooling power and maximum *COP*:

The midpoint current is

$$I_{mid} = \frac{I_{mp} + I_{COP}}{2} = \frac{11.526A + 2.956A}{2} = 7.241A$$

Using Equation (7), the maximum cooling power is

$$\dot{Q}_{cmid} = n\left(\alpha T_c I_{mid} - \frac{1}{2} I_{mid}{}^2 R - K\Delta T\right)$$
$$= (128)\left[(400\times10^{-6}\,V/K)(288K)(7.241A) - \frac{1}{2}(7.241A)^2(0.01\Omega) - \left(6.08\times10^{-3}\,\frac{W}{K}\right)(25K)\right]$$
$$= 53.815W$$

Using Equation (14), the maximum power input is

$$\dot{W}_{nmid} = n\left[\alpha I_{mid}(T_h - T_c) + I_{mid}{}^2 R\right]$$
$$= (128)\left[(400\times10^{-6}\,V/K)(7.241A)(25K) + (7.241A)^2(0.01\Omega)\right] = 76.377W$$

Using Equation (16), the midpoint *COP* is

$$COP_{mid} = \frac{\dot{Q}_{cmid}}{\dot{W}_{nmid}} = \frac{53.815W}{76.377W} = 0.705$$

The required cooling power is

$$\dot{Q}_{req} = 630W$$

The number of TEC modules required is

$$N = \frac{\dot{Q}_{req}}{\dot{Q}_{cmid}} = \frac{630W}{53.8W} = 11.7$$

Table E-2 Summary of the Results

	Max. Cool. Power	*Max. COP*	*Midpoint*
Current	I_{mp} = 11.526 A	I_{cop} = 2.956 A	I_{mid} = 7.241 A
Cooling power	Q_{cmp} = 65.576 W	Q_{cop} = 18.557 W	Q_{cmid} = 53.815 W
Power input	W_{cnp} = 184.8 W	W_{ncop} = 14.964 W	W_{nmid} = 76.377 W
COP	COP_{mp} = 0.355	COP_{max} = 1.24	COP_{mid} = 0.705
Number of modules	N_{mp} = 9.6	N_{cop} = 33.9	N_{mid} = 11.7
Design comments	Uneconomical (Too high power consumption)	Uneconomical (Too many modules)	Economical (reasonable design)

Comments

The results in Table E-2 are reflected in the *COP* and Q_c versus current curves (Figure E-2-2) plotted using Equations (7), (14), and (16) as a function of current with the material properties and inputs given in the example description. It is graphically seen in Figure E-2-2 that the maximum cooling power accompanies the very low *COP*, while the maximum *COP* accompanies very low cooling power. These lead to the uneconomical results. The midpoint of current between the maximum *COP* and maximum cooling power gives reasonable values for both. Automotive air conditioners intrinsically demand both a high *COP* and a high cooling power.

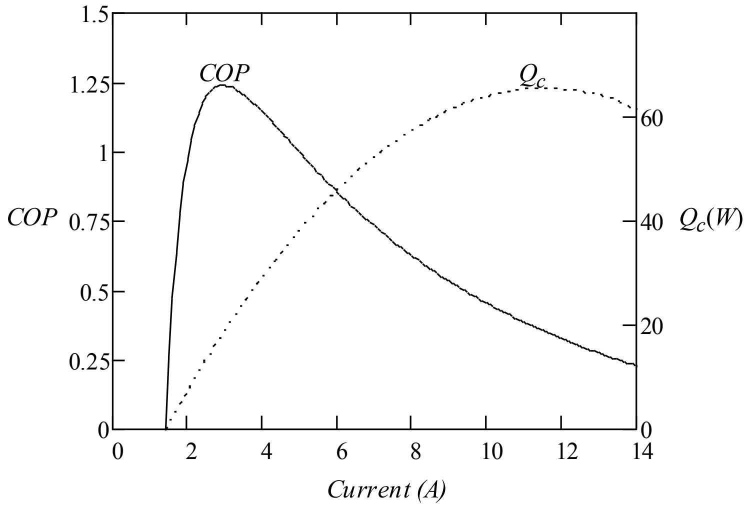


Figure E-2-2. *COP* and Q_c versus current for the given properties and inputs.

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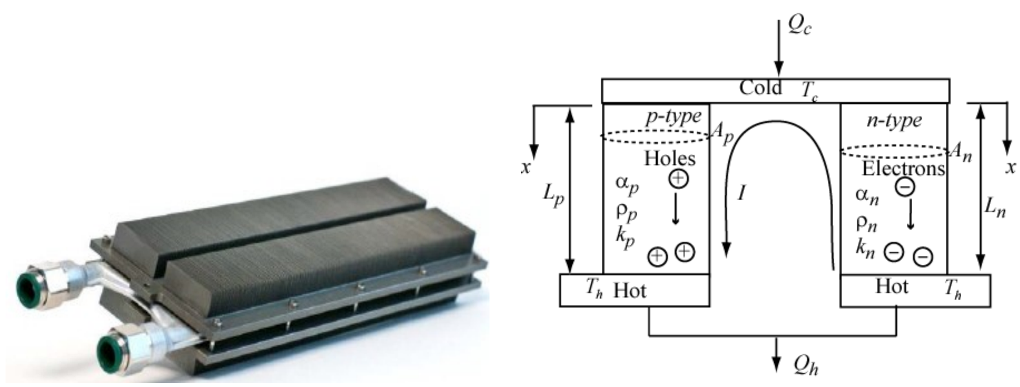
Problem P-2

A compact thermoelectric air conditioner is developed as an ambitious green energy project. N = 20 thermoelectric modules are installed between two heat sinks as shown in Figure P-2a. The module has n = 127 thermocouples, each of which consists of p- and n-type thermoelements as shown in Figure P-2b. Cabin air flows through the top and bottom heat sinks, while liquid coolant is routed through a heat exchanger at the center of the device wherein the coolant is cooled separately at the car radiator. With the effective design of both the heat sinks and heat exchanger, the cold and hot junction temperatures are maintained at 14 °C and 32 °C, respectively. Nanostructured thermoelectric properties of bismuth telluride based are given as $\alpha_p = -\alpha_n = 238$ μV/K, $\rho_p = \rho_n = 1.23 \times 10^{-3}$ Ωcm, and $k_p = k_n = 0.945 \times 10^{-2}$ W/cmK. The cross-sectional area A and pellet length L are 1 mm^2 and 1.1 mm, respectively. Answer the following questions for the whole air conditioner (Use hand calculations).

(a) For the maximum cooling power, compute the current, cooling power, and *COP*.
(b) For the maximum *COP*, compute the current, cooling power, and *COP*.
(c) If the midpoint of the current between the maximum cooling power and maximum *COP* is used for the optimal design, compute the current, the cooling power and *COP*.
(d) Draw the *COP*-and-cooling-power-versus-current curves with the given properties and information (Use Mathcad only for this part). Briefly explain the design concept.



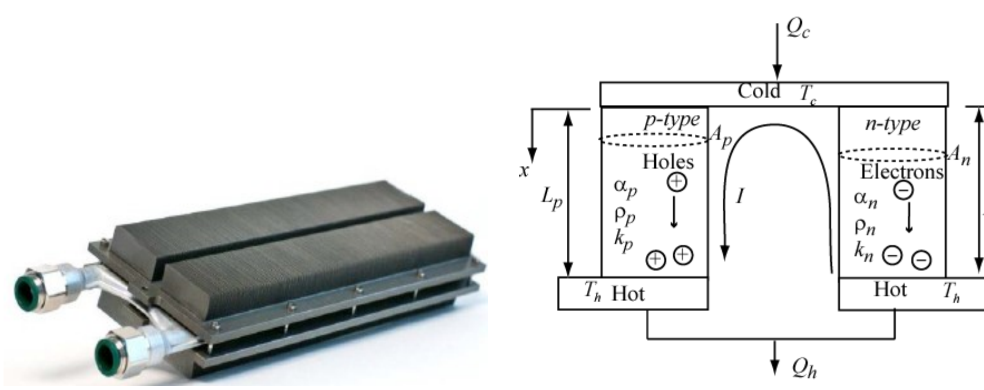
(a) (b)

Figure P-2. (a) A thermoelectric air conditioner. (b) A p-type and n-type thermocouple

Problem P-2-2

A compact thermoelectric air conditioner is developed as an ambitious green energy project. N = 40 thermoelectric modules are installed between two heat sinks as shown in Figure P-2-2 (a). The module has n = 127 thermocouples, each of which consists of p- and n-type thermoelements as shown in Figure P-2-2 (b). Cabin air flows through the top and bottom heat sinks, while liquid coolant is routed through a heat exchanger at the center of the device wherein the coolant is cooled separately at the car radiator. With the effective design of both the heat sinks and heat exchanger, the cold and hot junction temperatures are maintained at 15 °C and 30 °C, respectively. It is found that a commercial module of bismuth telluride is appropriate for this purpose, which has the maximum parameters: cooling power of 34.3 W, temperature difference of 67 °C, current of 3.9 °C, and voltage of 14.4 V. The cross-sectional area A and pellet length L are 1 mm^2 and 1.25 mm, respectively. Answer the following questions for the whole air conditioner (Use hand calculations).

(a) Obtain the effective material properties: the Seebeck coefficient, electrical resistance, and thermal conductivity.
(b) For the maximum cooling power, compute the current, cooling power, and *COP*.
(c) For the maximum *COP*, compute the current, cooling power, and *COP*.
(d) If the midpoint of the current between the maximum cooling power and maximum *COP* is used for the optimal design, compute the current, the cooling power and *COP*.
(e) Draw the *COP*-and-cooling-power-versus-current curves with the given properties and information (Use Mathcad only for this part). Briefly explain the design concept.



(a)　　　　　　　　　　　　　　(b)

Figure P-2-2. (a) A thermoelectric air conditioner. (b) A p-type and n-type thermocouple